UNIT I.

The subject theory of machines may be defined as that branch of Engineering - Science, which deals with the study of relative motion between the various parts of machine, and forces which act on them.

Machine:- A machine is a device which receives energy in some available form and utilises it to do some particular type of work.

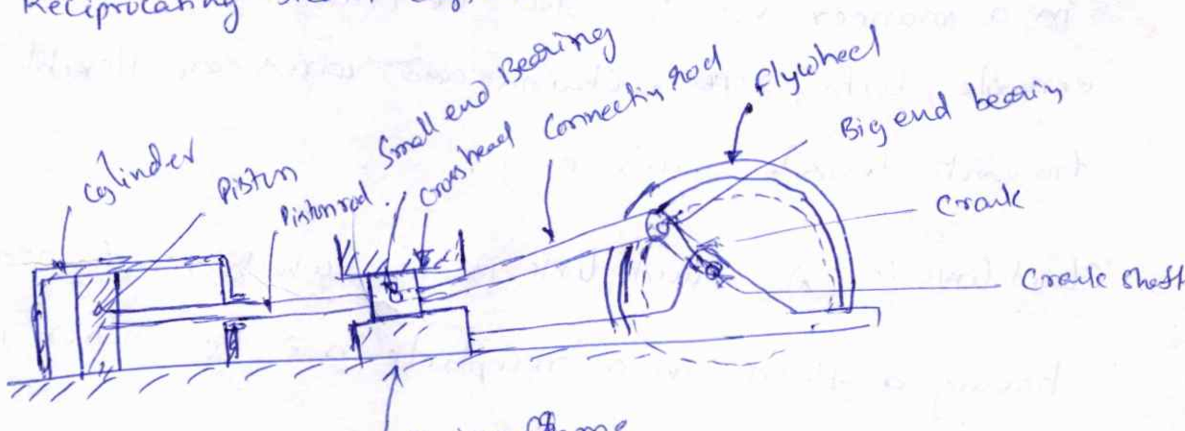
(or)

A machine is a device which receives energy and transforms it into some useful work.

Kinematic Link (or) Element:- Each part of the machine, which moves relative to some other part, is known as a Kinematic Link.

A Link may consists of several parts, which are rigidly fastened together, so that they do not move relative to one another.

Example:- Reciprocating Steam Engine



1st link :- Piston, Piston rod, and cross head constitutes one link.

2nd link :- Connecting rod, small end bearing, Big end bearing.

3rd link :- Crank, Crank shaft and fly wheel.

4th link :- Cylinder, Engine frame and Main Bearings.

A Link (or) element need not to be a rigid body, but it must be a resistant body. A body is said to be resistant body if it is capable of transmitting the required forces with neglegible deformation.

the characteristics of link
(or)
Properties

(i) It should have relative motion.
(ii) It must be a resistant body.

TYPES OF LINKS

In order to transmit motion, the driver and follower may be connected by the following three types of Links.

1. Rigid Link:- A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking rigid links donot exist.

2. Flexible link:- A flexible link is one which is partly deformed by in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link:- A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only as in the case of hydraulic presses, jacks and brakes.

STRUCTURE :- It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carring loads having straining action.

Machine	Structure
(1) The parts of machine move relative one another	(1) The parts of structure does not move relative to one another
(2) The machine transforms the available energy in to work	(2) The structure dont transform the energy in to work
(3) The machine transmits the motion and power	(3) The structure transmits only forces
(4) the parts of machine are called links	(4) The parts of structure are called the members
(5) The Machine constitutes the mechanism	(5) The structure is not having the mechanism
(6) Examples of machines are ;- Lathe, Drilling machine, Shaping Machine etc.	(6) Examples of structures are :- Bridges, Trusses, frames, Girders etc

Kinematic Pair :- The two links or elements of a machine, when in contact with each other are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e in a definite direction), the pair is known as kinematic pair.

Types of Constraint Motion

(1) Completely Constraint Motion (2) Incompletely Constraint Motion
(3) Successfully Constraint Motion

Kinematic chain:- When the Kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e completely or successfully constrained motion), it is called a Kinematic Chain.

(or)

In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

Mechanism:- When one of the links of a kinematic chain is fixed, then the chain is known as mechanism.

If the kinematic chain contains four links then it is called simple kinematic chain and the machine is called the simple machine.

If the kinematic chain contains more than four links, then it is called compound kinematic chain and the machine is called the compound machine

classification of kinematic Pairs

(1) According to the type of relative motion between the elements

(a) sliding pair (b) Turning Pair (c) Rolling pair (d) Screw pair (e) Spherical pair

(2) According to the type of contact between the elements

(a) Lower pair (b) Higher pair

(3) According to the type of closure

(a) self closed pair (b) force closed pair

movability (or) Number of degrees of freedom

In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom of the mechanism. It is defined as the number of input parameters (variables) which must be independently controlled in order to bring the mechanism into useful engineering purpose.

Now let us consider a plane mechanism with 'l' number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be $(l-1)$ and the total number of degrees of freedom will be $3(l-1)$ before they connected to any other link.

In general a mechanism with 'l' number of links connected by 'j' number of binary joints or lower pairs (having single degree of freedom) and 'h' number of higher pairs (having two degree of freedom), then the movabity (or) of the mechanism is given by the equation is

$$\boxed{\text{no. of DOF} \Rightarrow (n) \quad n = 3(l-1) - 2j - h}$$

for plane mechanisms $h = 0$

The above equation is called "KUTZ BATCH" criterion for movability of a mechanism having plane motion.

where l = no. of links

j = no. of Binary joints

h = no. of Higher pairs.

if $n=0$ it is called structure; $n=+1, \ldots$ etc it is a mechanism with no. of DOF.

$n=-1$ or negative it is called statistically indeterminant structure.

Grubler's Criterion for plane Mechanisms

The Grubler's criterion applies to mechanism with only single degree of freedom joints where the overall movability of the mechanism is Unity. Substituting $n=1$ and $h=0$ in Kutzbach equation ($n = 3(l-1)-2j-h$)

we have

$$1 = (3l-1) - 2j$$

$$\Rightarrow 1 = 3l - 3 - 2j$$

$$\Rightarrow \boxed{3l - 2j - 4 = 0} \longrightarrow$$ This equation is called the Grubler's criterion for plane mechanism

* Note:- A plane mechanism with a movability of 1 and only single degree of freedom joints can't have odd number of links. The simplest ~~form~~ possible mechanism of this type are a 4 bar mechanism and a slider crank mechanism in which $l=4$ and $j=4$.

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(1) The Relation between the number of links and number of kinetic pairs is given by the equation

$$\boxed{l = 2p - 4}$$

(2) The Relation between the number of joints and the number of links is given by the equation

$$\boxed{j = \frac{3}{2}l - 2}$$

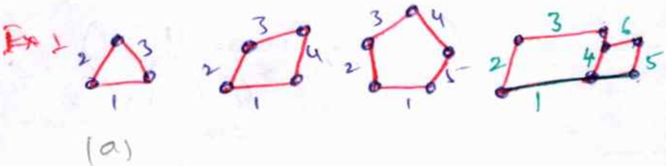
Note:- The above two equations are used for lower pairs to decide the Kinematic chain or not.

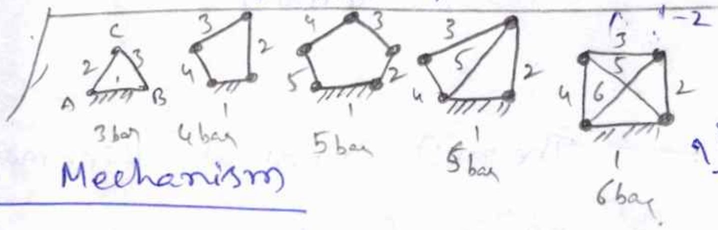
if (1) L.H.S > R.H.S → it is called Structure (or) Locked chain

(2) L.H.S = R.H.S → it is called a Kinematic chain having contrained motion

(3) L.H.S < = R.H.S → it is called a chain with Unconstrained motion

Ex:





Inversion of Mechanism

Def. The method of obtaining different mechanisms by fixing different links in a Kinematic chain is Known as 'inversion of the Mechanism'.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to fixed link) may be changed drastically.

Note :- The part (link or element) of mechanism which is initially moves with respect to the frame or fixed link is called driver and the part of mechanism to which motion is transmitted is called follower. Most of mechanisms are reversible i.e the same link can play the role of a driver and follower at different times.

Ex:- Reciprocating Steam engine → piston is a driver and flywheel is follower.

Reciprocating Compressor → piston is a follower and flywheel is driver.

TYPES OF KINEMATIC CHAINS

Simple Kinematic Chain (Having '4' links)

Complex (or) Compound Kinematic chain (Having more than '4' links)

The most important kinematic chains are those which consists of four lower pairs, each pair being a sliding pair or turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view.

(1) Four bar chain (or) Quadric cycle chain.

(2) Single Slider Crank Mechanism.

(3) Double Slider Crank Mechanism.

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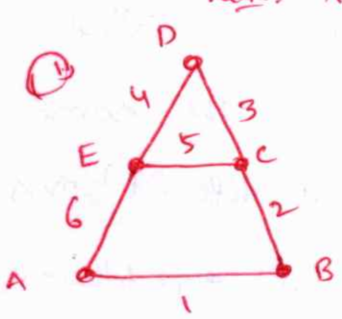
* *

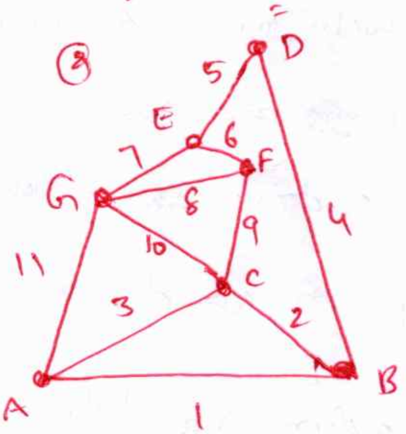
To Determine Nature of Chain (Structure, Kinematic chain, Unconstraint Chain)

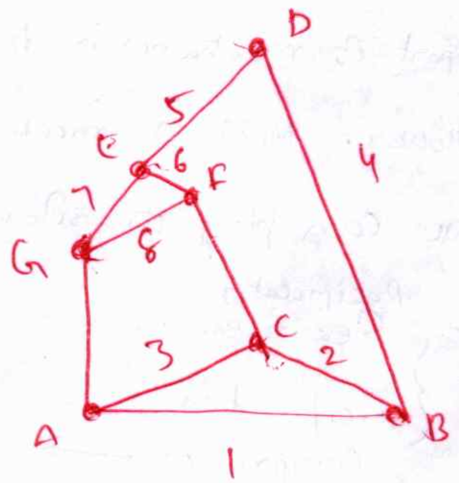
A.W. Klein is given $j + \frac{h}{2} = \frac{3}{2}l - 2$

j = no. of binary joint
h = No. of higher pairs
l = no. of links

Note:- for Lower pairs $h = 0$

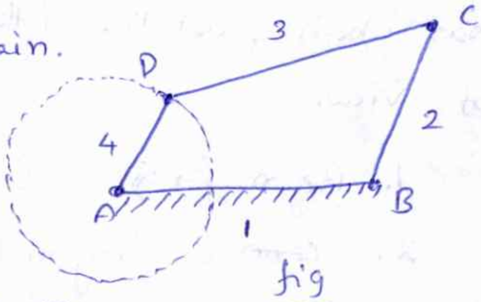






FOUR BAR CHAIN (or) Quadric Cycle Chain

The simplest and the basic kinematic chain is a four bar chain or a quadric cycle chain. It consists of a four links each of them forms a turning pairs at A, B, C and D. The four links may be of different lengths.



fig

According to the Grashof's law for four bar mechanism, the sum of shortest and longest link lengths should not be greater than the sum of the remaining two link lengths, if there is to be continuous relative motion between the two links.

A very important consideration in designing a mechanism is to ensure ~~that the~~ input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be usefull. In four bar chain, one of the links in particular the shorter link will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such link is known as a crank or driver(4). In the fig the link 4 is AD ~~the~~ called the crank. The link 'BC' makes a partial rotation or oscillates is known as lever(2) or rocker or follower. The link 'CD' which connects the crank and lever is called connecting rod(3) or coupler. The fixed link 'AB'(1) is known as frame(1) of the mechanism.

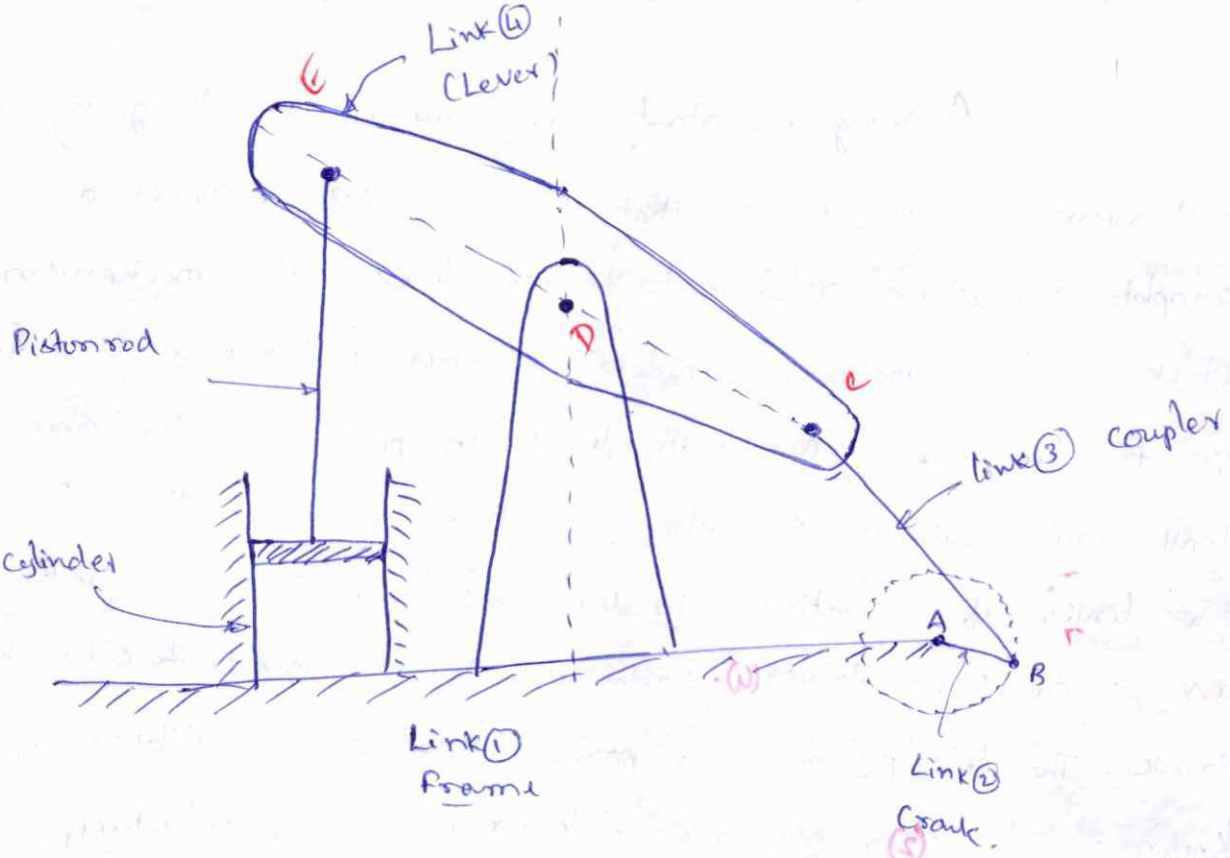
When the crank (AD) is a driver, the mechanism is transforming rotary motion into oscillating motion.

Inversions of four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view.

1. Beam Engine (crank and lever Mechanism)
2. ~~com~~ Coupling rod of a locomotive (Double crank mechanism)
3. Watt's indicator mechanism (Double lever mechanism)

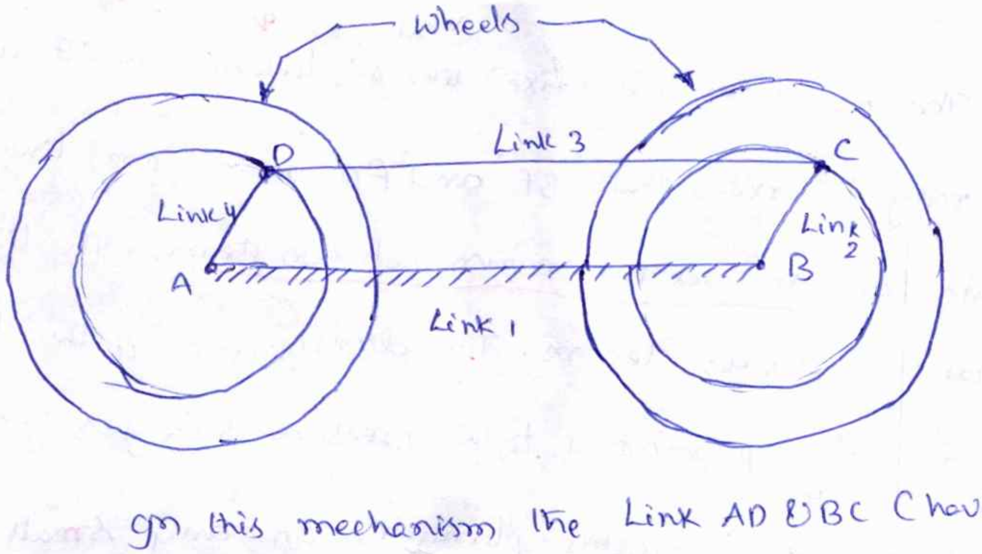
1. Beam Engine



A part of the mechanism of beam engine which consists of four links a is shown in fig. In this mechanism when the crank rotates about the fixed centre A, and the lever oscillates about a fixed centre D. The End 'E' of the lever 'CDE' is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion in to reciprocating motion.

2. Coupling Rod of a Locomotive

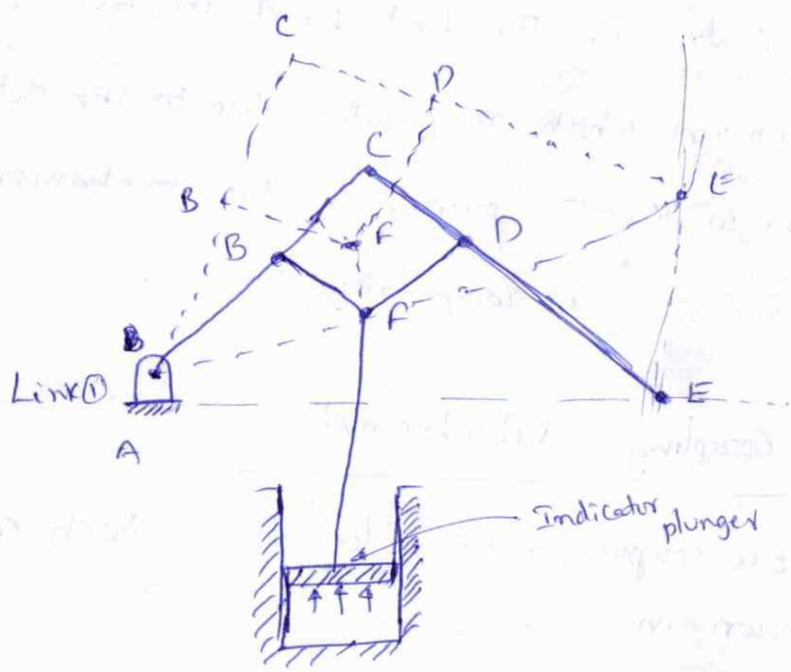
The mechanism of a coupling rod of a locomotive which consists of 'Four' links is shown in fig.



In this mechanism the Link AD & BC (having equal lengths) acts as cranks and are connected to the respective wheels. The Link 'CD' acts as a coupling rod and the link 'AB' is fixed in order to maintain a constant center to center distance between them. This mechanism is meant for transmitting rotary motion from one wheel to other wheel.

3. Watt's indicator Mechanism

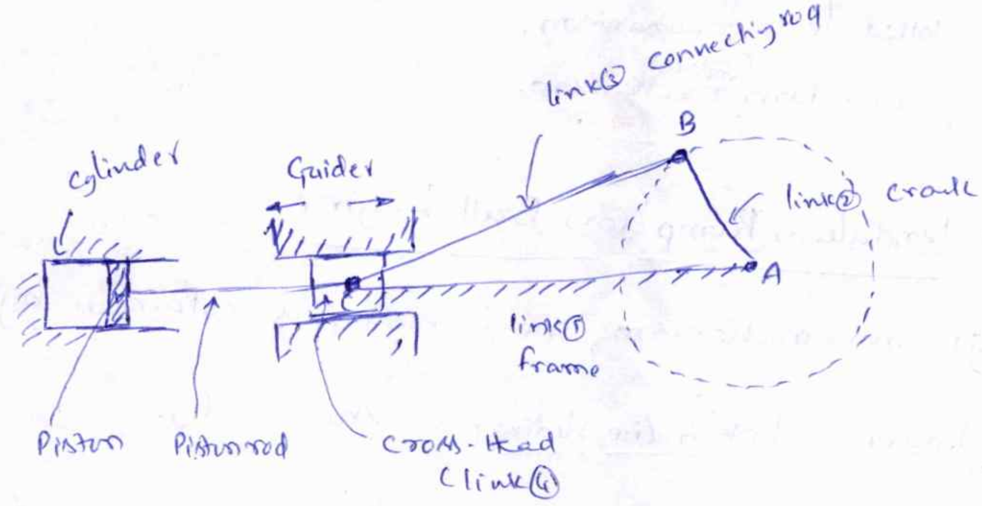
A watt's indicator mechanism which consists of '4' links is shown in fig. This mechanism also called as watt's straight line mechanism.



The four links are fixed link at 'A', link AC, link CE and link 'BFD'. It may be noted that 'BF' and 'FD' forms one link because these two parts have no relative motion between them. The links CE and BFD acts as levers. The displacement of the link 'BFD' is directly proportional to the pressure of the gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point 'E' at the end of the link 'CE' traces out approximately a straight line.

Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is usually found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice-versa.



Single Slider Crank Chain

In a slider crank chain as shown in fig. the links 1&2, links 2&3, links 3&4 form the three turning pairs, while the link 4&1 forms a sliding pair.

The link ① corresponds to the frame of the engine, which is fixed, The link ② corresponds to the crank, The link ③ corresponds to the connecting rod and the link ④ corresponds to cross head. As the crank rotates, the cross head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain

The following are the important inversions of the mechanism found.

(1) Pendulum Pump (or) Bull engine.

(2) Oscillating cylinder engine.

(3) Rotary internal Combustion engine (or) Gnome engine.

(4) Crank and Slotted lever quick return motion mechanism.

(5) Whitworth quick return motion mechanism.

(1) Pendulum Pump (or) Bull Engine

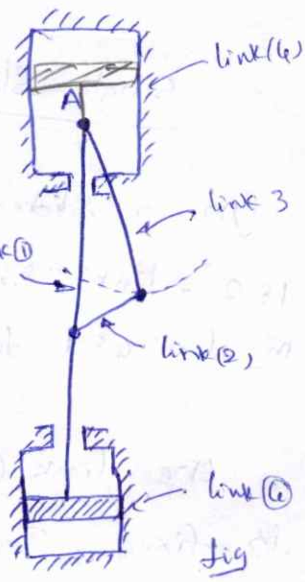
In this mechanism, the inversion is obtained by fixing the cylinder or link '4' (i.e sliding pair) shown in fig.

Link ① → Piston Rod

Link ② → Crank

Link ③ → Connecting Rod

Link ④ → Cylinder



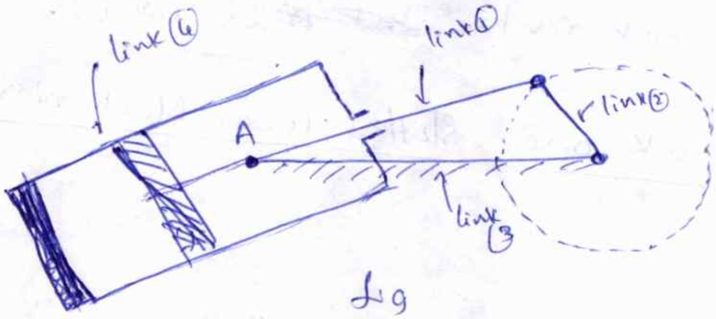
fig

In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to a fixed link '4' at A and the piston attached to the piston rod (link 1) reciprocates. The Duplex pump which is used to supply feed water to boilers have two pistons attached to link ① as shown in above fig.

(2) Oscillating Cylinder Engine

The arrangement of oscillating cylinder engine mechanism as shown in fig. is used to convert reciprocating motion into rotary motion.

① → Piston rod.
② → Crank
③ → Connecting Rod.
④ → Cylinder.

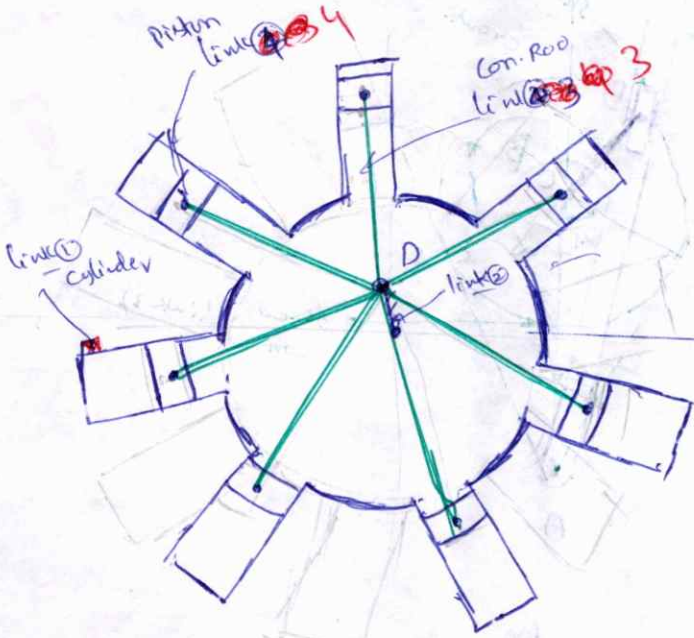


fig

In this mechanism the link ③ forming the turning pair is fixed. The link ③ corresponds to the connecting rod of reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to the piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

(3) Rotary Internal Combustion Engine (or) Gnome Engine

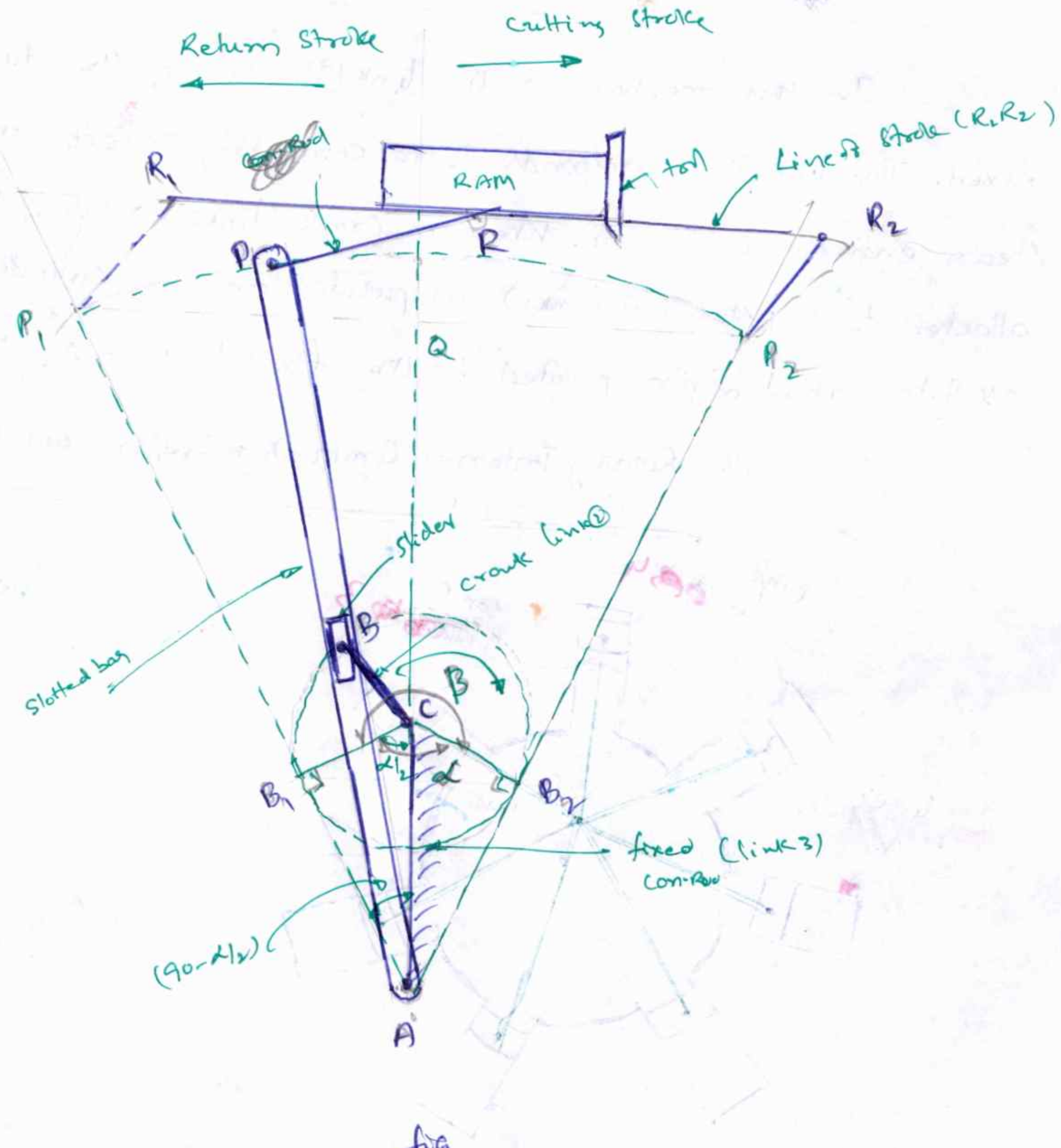
link ② crank fixed



Sometimes back, rotary I.C-engines wer Used in aviation.

But now a days gas turbines are Used in its place. It Consists of Seven cylinders in one plane and all revolves about fixed center 'D'. as shown in fig. while the Crank (link 2) is fixed. In this mechanism when the connectin rod (link 3) rotates, the piston (link 4) reciprocates inside the cylinder which revolves ~~about itself~~ (link 1).

(iv) Crank and Slotted lever Mechanism



fig

The Crank and Slotted lever mechanism mostly used in Shaping Machines, Slotting machines and in rotary I.C. engines.

In this mechanism the link 'AC' [corresponds the con. rod of reciprocating steam engine] (i.e link 3) forming the turning pair is fixed and shown in fig. The driving crank (CB) (link 2) revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B which slides along the slotted bar 'AP' and thus causes 'AP' to oscillate about the pivot point 'A'. A short link 'PR' transmits the motion from 'AP' to the 'ram' which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (i.e R_1R_2) is perpendicular to 'AC' produced.

In the extreme positions AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 through an angle 'β'. in clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (through an angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360-\beta} \text{ (or) } \frac{360-\alpha}{\alpha}$$

Since the tool travels a distance of R_1R_2 during the cutting and return stroke, therefore travel of tool (or) length of stroke. $= R_1R_2 = P_1P_2 = 2P_1Q$.

from fig. $2P_1Q = 2 \times \sin(90-\alpha/2)\, AP_1 = 2AP_1 \cos\alpha/2 = 2AP \times \cos\alpha/2$

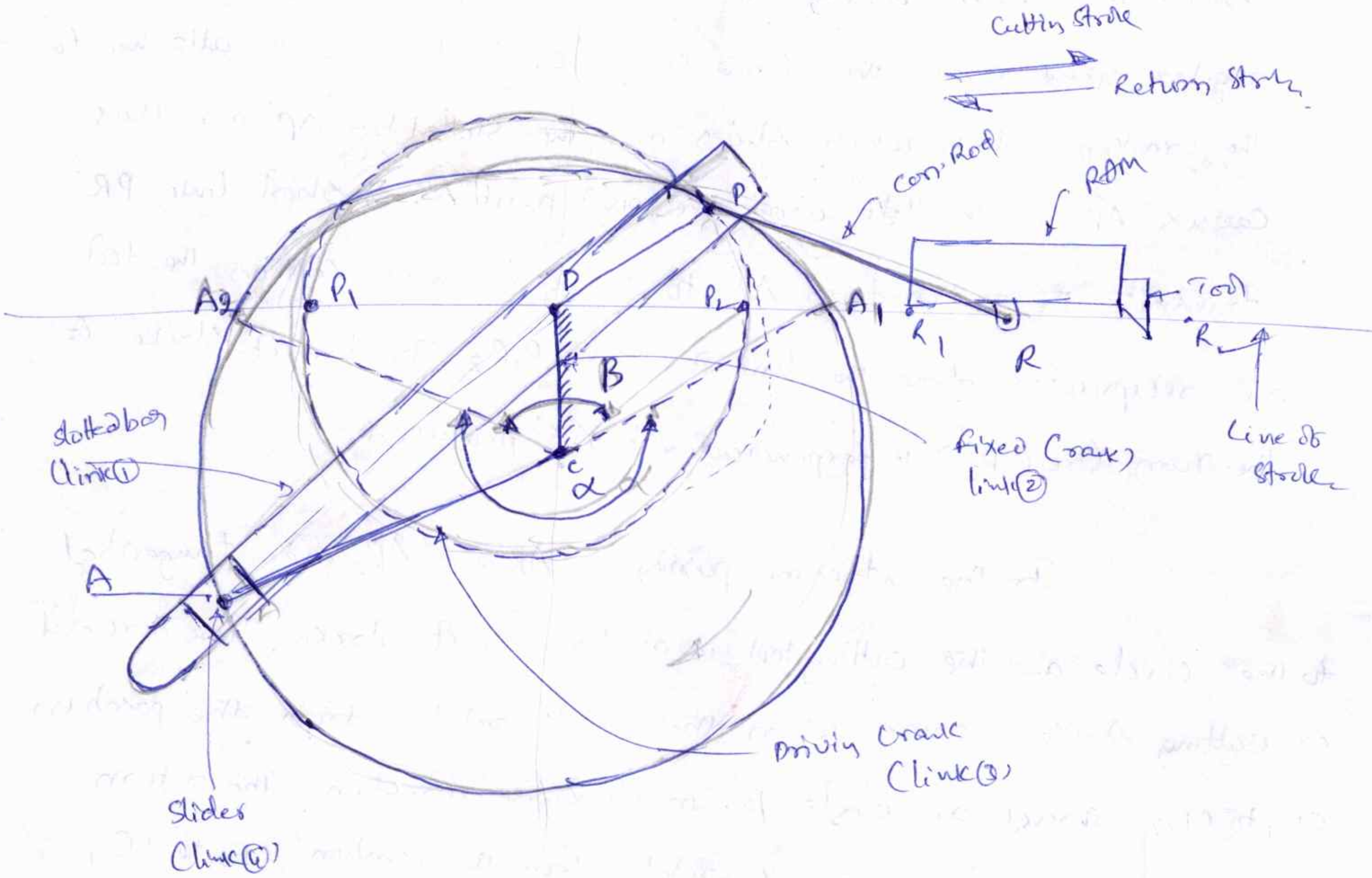
$[\because AP_1 = AP_2 = AP]$

$\Rightarrow R_1R_2 = 2P_1Q = 2AP \times \frac{CB_1}{AC}$

$$\Rightarrow \boxed{R_1R_2 = 2AP \times \frac{CB}{AC}}$$

WHITWORTH QUICK RETURN MOTION MECHANISM

This mechanism is mostly used in Shaping and Slotting machin.
The mechanism is shown in fig.



In this mechanism the link 'CD' (link 2) forming the turning pair is fixed. The link '2' corresponds to crank in reciprocating steam engine. The driving crank CA (link 3) rotates at angular speed. The slider (link 4) attached to the crank pin at 'A' slides along the slotted bar PA (link 1) which oscillates at a pivot point D. The connecting rod PR carries the Ram at 'R' to which a cutting tool is fixed. The motion of a tool is contrained along the line 'RD' produced i.e along a line passing through 'D' and perpendicular to 'CD'.

When the driving crank 'CA' moves from CA_1 to CA_2 (or the link 'DP' from the position DP_1 to DP_2) through an angle 'α' in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance '2PD'.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link 'DP' from DP_2 to DP_1) through an angle 'β' in the clockwise, the tool moves back from the right hand end of its stroke to the left ~~end~~ hand end.

A little consideration will show that the time taken during the left to right movement of the ram will be equal to the time taken by the driving crank to move from CA_1 to CA_2.
Similarly, the time taken during the right to left movement of the ram will be equal to the time taken by the driving crank move from CA_2 to CA_1.

Since the crank link 'CA' rotates at uniform angular velocity therefore time taken during the cutting stroke is more than the time taken during the return stroke.

$$\therefore \frac{\text{Time of Cutting Stroke}}{\text{Time of Return Stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha} = \frac{360-\beta}{\beta}$$

The length of effective cutting stroke $R_1R_2 = P_1R_1 = P_2R_2 = PR = \underline{2 \times PD}$

$$\boxed{R_1R_2 = 2PD}$$

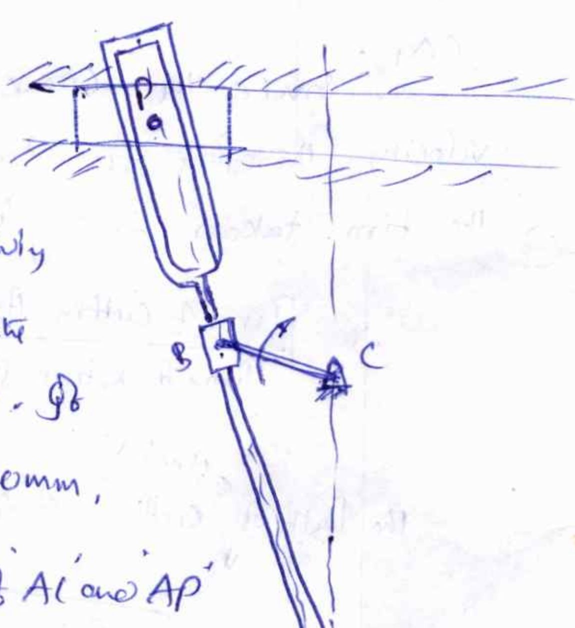
Problems

① A crank and slotted lever mechanism used in a shaper has a center distance of 300 mm between the center of oscillation of the slotted lever and the center of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke

② In a crank and slotted lever quick return mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in extreme position and the time ratio of cutting stroke to return stroke.

If the length of slotted bar is 450 mm, find the length of stroke, if the length of stroke passes through the extreme positions of the free end of the lever.

③ The fig shows the lay out of a quick return mechanism of the oscillating link type, for a special purpose machine. The driving crank 'BC' is 30 mm long and the ratio of the working stroke to the return stroke is to be 1.7. If the length of the working stroke of ($R_1 R_2$) is 120 mm,

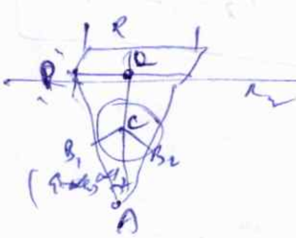


Determine the length of the dimensions of 'AC' and 'AP'

(Ans:- $\alpha = 133.3^\circ$

AC = 75.7 mm

AP = 151.4 mm

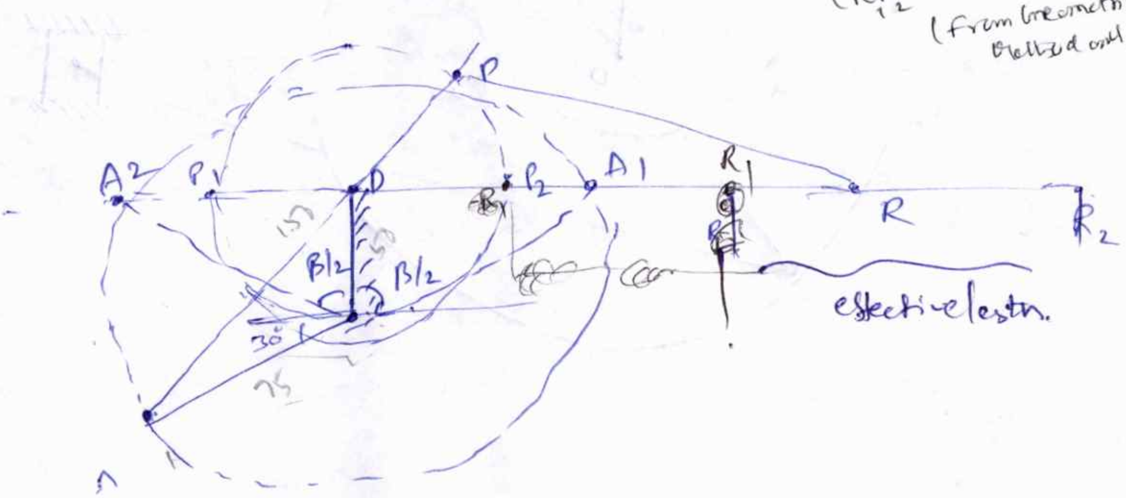


(4) In a whitworth quick return motion mechanism as shown in fig the distance between the fixed centers is 50mm and the length of the driving crank is 75mm. The length of slotted lever is 150mm and the length of connecting rod is 135mm. find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

($\beta = 96.4°$)

$\frac{t_c}{t_R} = 2.735$

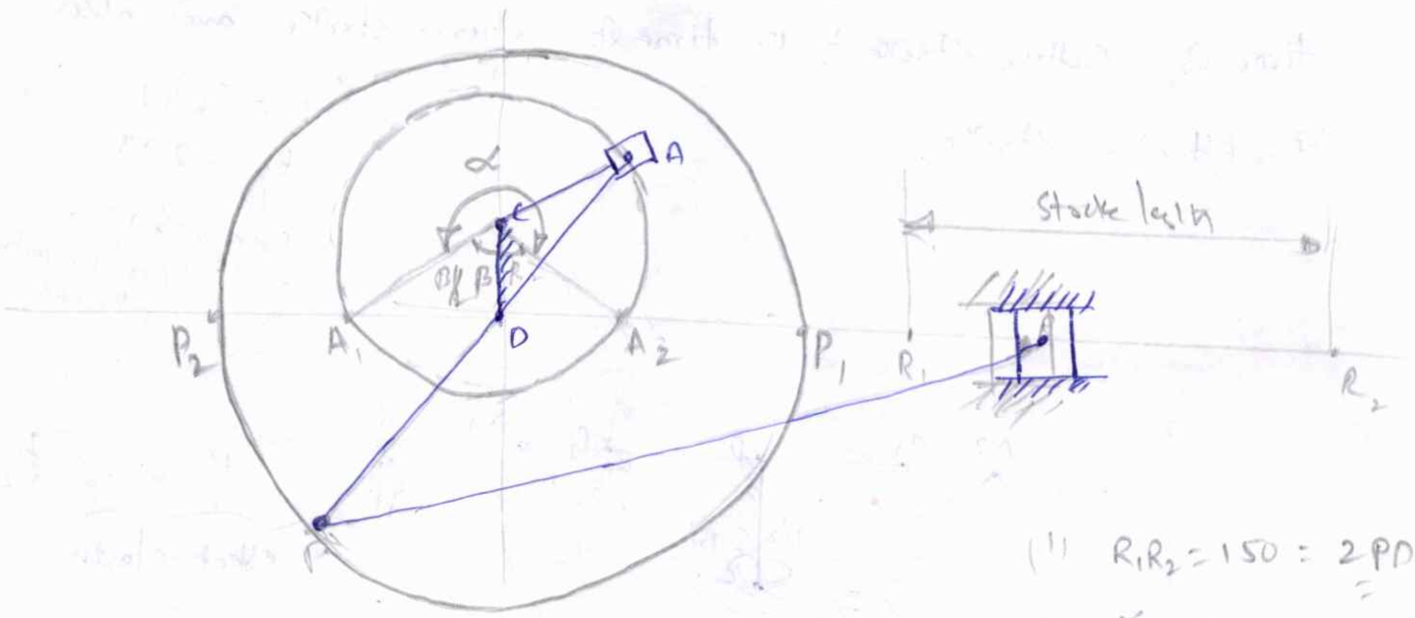
($R_1R_2 = 87.5$ mm, (from Geometrical method only))



(5) The whitworth quick return motion mechanism has the driving crank 150 mm long. The distance between the fixed centers is 100mm. the line of stroke of the ram passes through the centre of rotation of the slotted lever whose free end is connected to the ram by a connecting link. find the ratio of time of cutting to time of return

(Ans: 2.735)

(6) A Whitworth quick return motion mechanism, as shown in fig has the following particulars. Length of stroke = 150mm; Driving crank length = 40mm; Ratio of cutting time to return time = 2; find the lengths of CD and PD. Also determine the angles α and β.



(1) $R_1R_2 = 150 = 2PD$

(2) $\frac{\alpha}{\beta} = 2$

(3) $CA = 40$

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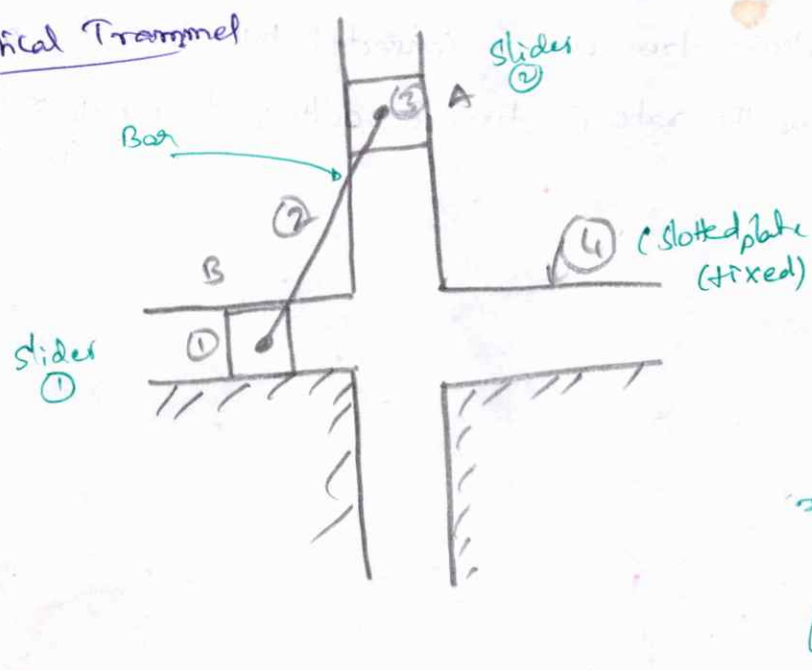
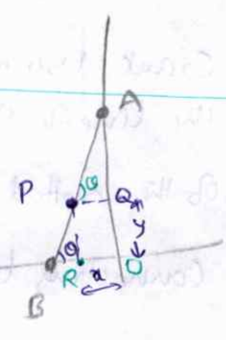
(1) Elliptical Trammel





from Δ PQA

$\cos\theta = \frac{PQ}{AP} = \frac{x}{AP}$

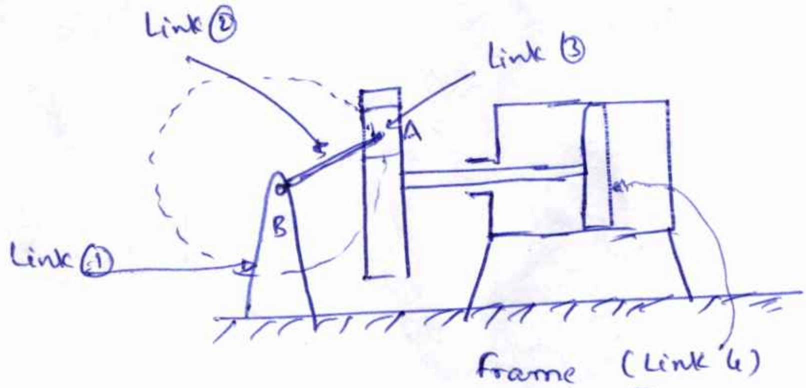
from the Δ BRP

$\sin\theta = \frac{BR}{BP} = \frac{y}{BP}$

$\sin^2\theta + \cos^2\theta = 1$

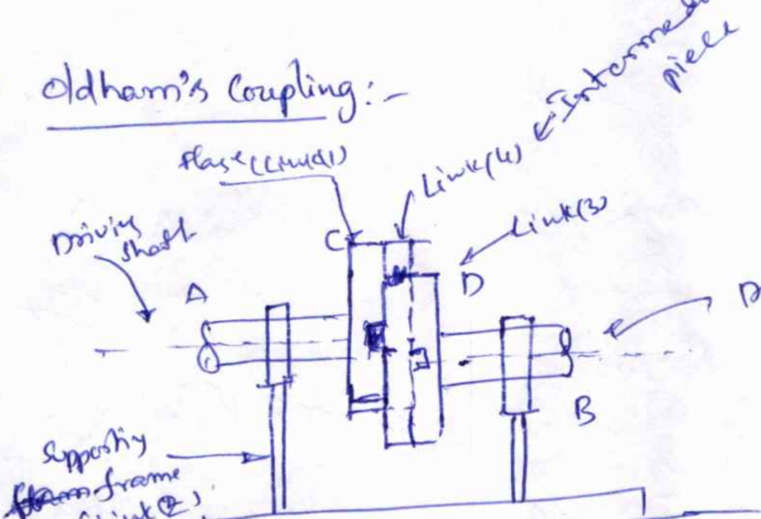
$$\boxed{\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1}$$

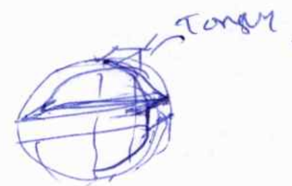
(2) Scotch yoke mechanism



This mechanism is used for converting rotary motion into reciprocating motion. The inversion is obtained by fixing either the Link ① or link ③. In the fig the link ① is fixed. In this mechanism when the link 2 rotates about center B, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

(3) Oldham's Coupling:-





∴ maximum sliding speed of each tounge (m/sec)

$v = \omega \cdot r$

ω = Angular speed of each shaft

r = Distance between the two axes of shaft

Types of joints in a chain

1. Binary Joint :- When

Unit II :- (Mechanisms with Lower Pairs)

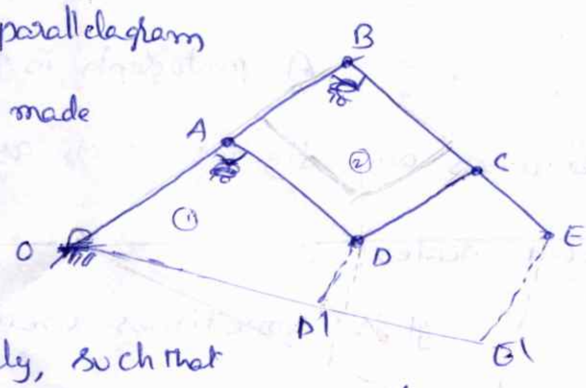
§ STRAIGHT LINE MOTION MECHANISMS

When the two elements of a pair have a surface contact and the relative motion takes place, the surface of one element slides over the surface of the other, the pair formed is known as lower pair.

PANTOGRAPH

A pantograph is an instrument used to produce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.

It consists of a jointed parallelogram 'ABCD' as shown in fig. It is made up of bars connected by turning pairs. The bars 'BA' and 'BC' are extended to 'O' and 'E' respectively, such that



fig

$$\frac{OA}{AB} = \frac{OB}{BE} \Rightarrow \boxed{\frac{OA}{OB} = \frac{AD}{BE}}$$

Thus for all relative positions of the bars, the triangles OAD, and OBE are similar and the points 'O, D, E' are in one straight line. It may be proved that point 'E' traces out the same path as described by the point 'D'.

From similar triangles 'OAD' and 'OBE', we find that

$$\boxed{\frac{OD}{OE} = \frac{AD}{BE}}$$

Let point 'O' be fixed and the points 'D' and 'E' move to some new position D' & E'. Then

$$\frac{OD}{OE} = \frac{DD'}{EE'}$$

A little consideration will show that the straight line DD' is parallel to the to the straight line EE'. Hence if 'O' is fixed to the frame of machine by means of turning pair and 'D' is attached to a point in the machine, which has rectilinear motion relative to the frame, then 'E' will also trace out a straight line path. Similarly, if 'E' is constrained to move in a straight line, then 'D' will trace out a straight line parallel to the former.

A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc. on enlarged or reduced scales.

It is sometimes used as an indicator rig in order to reproduce to a small scale the displacement of the cross-head and therefore of the piston of reciprocating steam engine.

It is also used to guide cutting tools.

Straight line Motion Mechanisms

One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called straight line mechanisms.

TYPES

The straight line motion mechanisms are two types

(1) The mechanisms in which only turning pairs are used.

(2) The mechanisms in which one sliding pair is used.

The above two mechanisms may produce exact straight line motion (or) approximate straight line motion.

① Exact straight Line Motion Mechanisms with Turning Pairs

The principle adopted for mathematically correct or exactly straight line motion is described in fig.

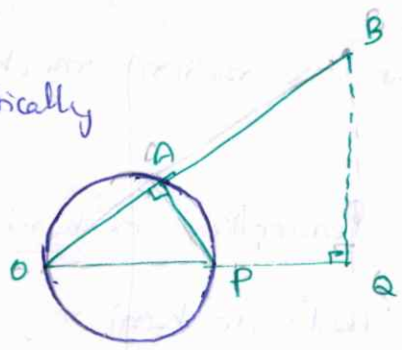


fig.

Let 'O' be the ~~fixed~~ point on the circumference of a circle of diameter 'OP'. Let 'OA' be any ~~ear~~ chord and 'B' is a point on 'OA' produced, such that

$$OA \times OB = \text{constant}.$$

then the locus of a point 'B' straight line perpendicular to the diameter 'OP'. This may be proved as follows

Draw 'BQ' perpendicular to 'OP' produced, join AP. The triangles OAP and OQB are similar

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

(or) $OA \times OB = OQ \times OP$

(or) $OQ = \dfrac{OA \times OB}{OP}$

But 'OP' is constant as it is the diameter of a circle, therefore, if $OA \times OB$ is constant, then 'OQ' will be constant. Hence the point 'B' moves along the straight path 'BQ', which is perpendicular to OP.

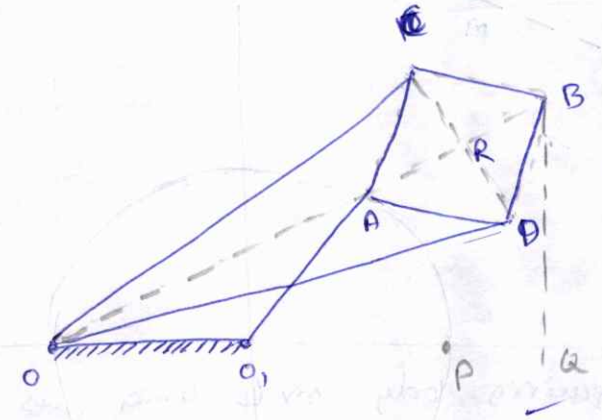
The following are the two well known types of exact straight line motion mechanisms made up of turning pairs.

(1) Peaucellier mechanism

(2) Hart's Mechanism

Peaucellier Mechanism

This mechanism contains eight links (8). It consists of a fixed link OO_1 and the other straight links O_1A, OC, OD, AD, CA, DB, BC are connected by turning pairs at their intersections as shown in fig. (ABCD → is forms a Rhombus)



The Pin at 'A' is constrained to move along the circumference of a circle with the fixed diameter 'OP' by means of a link O_1A.

from fig. $AC = CB = BD = DA$; $OC = OD$; and $OO_1 = O_1A$

It may be proved that the product $OA \times OB$ remains constant, when the link O_1A rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC

$$OC^2 = OR^2 + RC^2 \quad \text{—(i)}$$

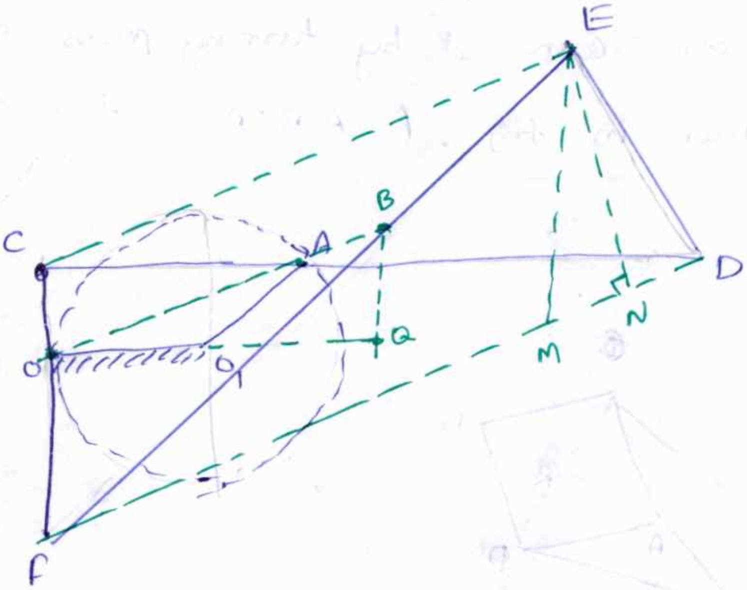
$$CB^2 = BR^2 + RC^2 \quad \text{—(ii)}$$

(i) − (ii) ⇒ $OC^2 - CB^2 = OR^2 - BR^2$ $\quad [a^2 - b^2 = (a+b)(a-b)]$

⇒ $OC^2 - CB^2 = (OR + BR)(OR - BR) = OB \times OA$ $\quad [\because BR = AR]$

⇒ ∴ $OA \times OB =$ Constant $\quad \{\because$ OC & CB are links$\}$

Hart's Mechanism



This mechanism requires only six (6) links as compared with eight(8) links required by the Peaucellier mechanism.

It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection as shown in fig. The links FC and DE are equal in length and the lengths of links CD and EF are also equal. The points O. A. and B devide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

Hence OAB is a straight line. It may be proved now that the product OA×OB is constant.

from similar triangles CFE and OFB

$$\frac{CE}{FC} = \frac{OB}{OF} \quad (\text{or}) \quad \boxed{OB = OF \times \frac{CE}{FC}} \quad \text{—(i)}$$

from similar triangles FCD and OCA.

$$\frac{OA}{OC} = \frac{FD}{CF} \text{ or } \frac{FD}{FC}$$

$$\Rightarrow \boxed{OA = OC \times \frac{FD}{FC}} \quad \text{—— (ii)}$$

multiplying (i) × (ii) $\Rightarrow OA \times OB = OC \times \frac{FD}{FC} \times OF \times \frac{CE}{FC}$

$$\Rightarrow OA \times OB = \frac{OC \times OF}{FC^2} \times FD \times CE$$

But $\frac{OC \times OF}{FC^2}$ = constant

Ita $\Rightarrow \boxed{OA \times OB = FD \times CE}$

Now drawing a parallel EM from point E to CF and EN perpendicular to FD. Therefore.

$$FD \times CE = (FN + ND) \times FM \qquad (\because CE = FM)$$

$$= (FN + ND) \times (FN - NM)$$

$$= (FN + ND) \times (FN - ND) \quad [\because NM = ND]$$

$$\boxed{FD \times CE = FN^2 - ND^2}$$

$\Rightarrow$ ~~But from right angle triangles FNM & FND~~ $\left\{ \begin{array}{l} \cancel{FM^2 = FN^2 + NM^2} \\ \cancel{FD^2 = FN^2 + ND^2} \end{array} \right.$ (NM) $\left[\because \begin{array}{l} NM \\ = ND \end{array}\right]$

$\therefore \cancel{FD \times CE = (FD^2 - NM^2) - (FM^2 - NM^2)} = \cancel{FD^2 - FM^2}$

From right angle triangle FNE & END

$$FE^2 = FN^2 + EN^2 \qquad ED^2 = EN^2 + ND^2$$

$$\Rightarrow FN^2 = FE^2 - EN^2 \qquad ND^2 = ED^2 - EN^2$$

$$\therefore \quad FD \times CE = FN^2 - ND^2 = (FE^2 - EN^2) - (ED^2 - EN^2)$$

$$\Rightarrow \quad FD \times CE = FE^2 - ED^2 = \text{constant}$$

$\therefore OA \times OB =$ Constant.

It therefore follows that if the mechanism is pivoted about 'O' as fixed point and the point 'A' is constrained to move on a circle with Centre O_1, then the point 'B' will trace a straight line perpendicular to the diameter OP produced.

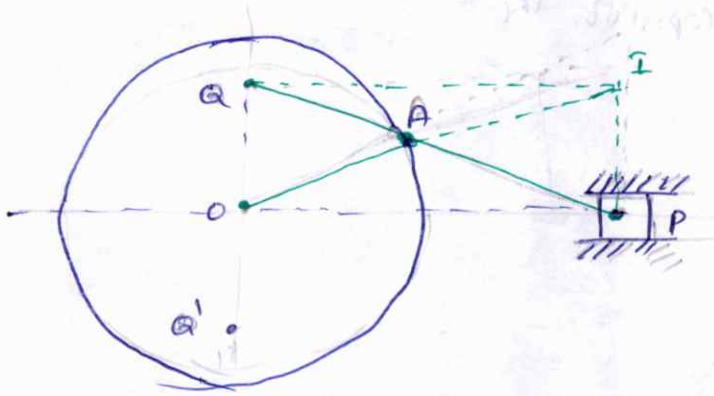
Note:- This mechanism has a great practical disadvantage that even when the path of 'B' is short, a large amount of space is taken up by the mechanism.

Exact Straight Line Motion Consisting of One Sliding Pair

This Mechanism contains one sliding Pair and remain turning Pairs.

Example of this type Mechanism is:- Scott-Russell's Mechanism

Scott-Russell's Mechanism



The Scott-Russell's Mechanism Consists of a fixed member and moving member 'P' of a sliding pair as shown in fig.

The straight link 'PAQ' is connected by turning pairs to the link 'OA' and the link (slider) P. The link 'OA' rotates about 'O'. A little Consideration will show that the mechanism 'OAP' is same as that of the reciprocating engine mechanism in which 'OA' is the Crank and 'PA' is the Connecting rod. In This mechanism, the straight line motion is not generated but it is merely Copied.

'A' is the middle point of 'PQ' and OA = AP = AQ. The instantaneous Center for the link PAQ lies at 'I' in 'OA' produced and is such that 'IP' is perpendicular to 'OP'. Joint IQ. Then 'Q' moves along the perpendicular to IQ.

Since 'OPIQ' is a rectangle and 'IQ' is ~~rectafle~~ perpendicular to OQ, therefore 'Q' moves along the vertical line 'OQ' for all positions of QP. Hence 'Q' traces the straight line OQ'. If 'OA' makes one complete revolution, then 'P' will oscillate along the line OP' through a distance '2OA' on each side of 'O' and 'Q' will oscillate along 'OQ'' through the same distance '2OA' above and below O.

Thus, the locus of Q is a copy of the locus of P.

Approximate Straight Line motion Mechanisms

The approximate straight line motion mechanisms are the modifications of the '4'- Bar Chain mechanism. The following are the important from subject point of view.

1. Watt's Mechanism.

2. Modified Scott - Russel Mechanism

3. Grasshopper Mechanism

4. Tchebicheff's mechanism.

5. Roberts Mechanism.

1. Watt's Mechanism

It is a crossed four bar chain mechanism and was used by 'Watt' for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.

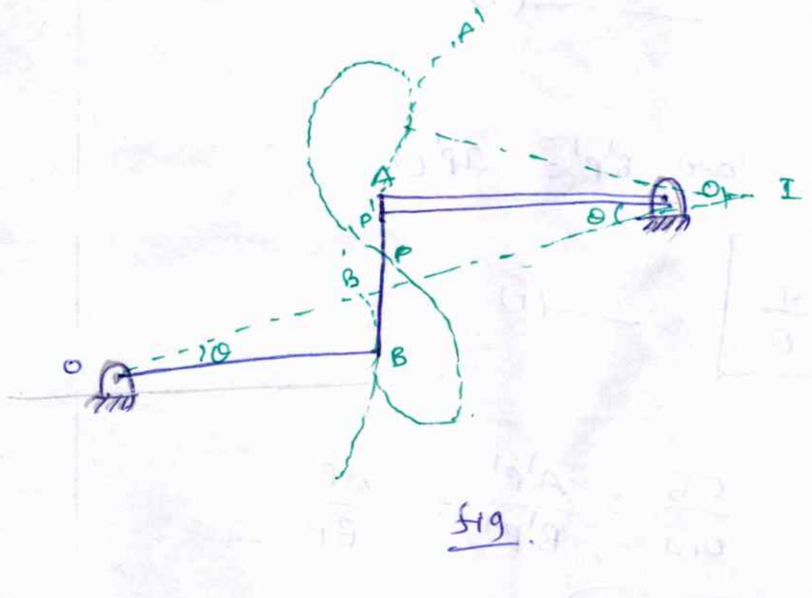


fig.

In fig 'OBAO₁' is a crossed four bar chain in which 'O' and 'O₁' are fixed. In the mean position of the mechanism, OB and O₁A are parallel and coupling rod 'AB' is perpendicular to

O_1A and OB. The tracing point 'P' traces out an approximate straightline over certain positions of its movement, if

$$\frac{PB}{PA} = \frac{O_1A}{OB}.$$ this may be proved as follows:

A little consideration will show that in the initial mean position of the mechanism, the instantaneous centers of the link 'BA' lies at infinity. Therefore the motion of the point 'P' is along the vertical line BA. Let $OB'A'O_1$ be the new position of the mechanism after the links 'OB' and 'O_1A' are displaced through an angle θ and ϕ respectively. The instantaneous center now lies at I. Since the angle 'θ' and 'ϕ' are very small, therefore

$$\text{Arc } BB' = \text{Arc } AA'$$

(or)

$$\Rightarrow \theta \times OB = O_1A \times \phi$$

$$\Rightarrow \frac{OB}{O_1A} = \frac{\phi}{\theta} \quad \text{——— (i)}$$

But Also $A'P' = IP'\phi$ and $B'P' = IP'\theta$

$$\Rightarrow \boxed{\frac{A'P'}{B'P'} = \frac{\phi}{\theta}} \quad \text{——— (ii)}$$

From (i) & (ii)

$$\Rightarrow \frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP}$$

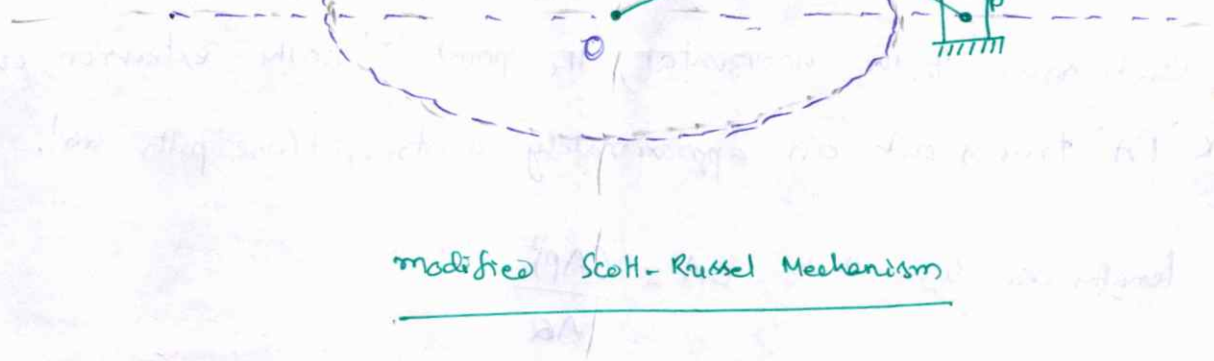
(or) $$\boxed{\frac{O_1A}{OB} = \frac{PB}{PA}}$$

Thus the point 'P' divides the link AB in to two parts whose lengths are inversely proportional to the lengths of the adjacent links.

2. Modified Scott-Russel Mechanism

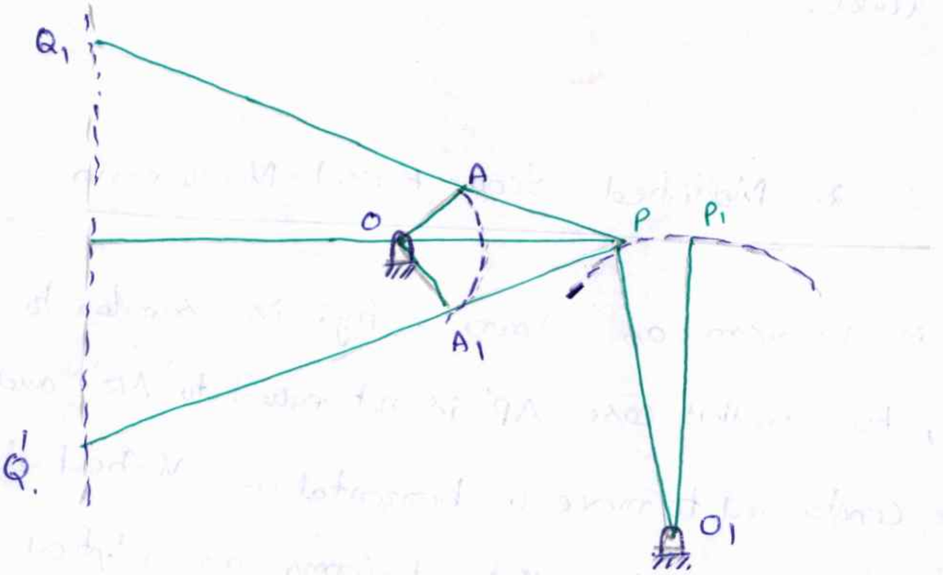
This Mechanism as shown in fig. is similar to Scott-Russel mechanism, but in this case 'AP' is not equal to 'AQ' and the points 'P' and 'Q' are constrained to move in horizontal and vertical directions. A little consideration will show that it forms an elliptical trammel, So that any point 'A' on 'PQ' traces an ellipse with semi-major axis 'AQ' and semi minor axis 'AP'.

If the point 'A' moves in a circle, then for point 'Q' to move along an approximate straight line, the length 'OA' must be equal to $\frac{(AP)^2}{AQ}$. This is limited to only small displacement of 'P'.



Modified Scott-Russel Mechanism

3. Grasshopper mechanism



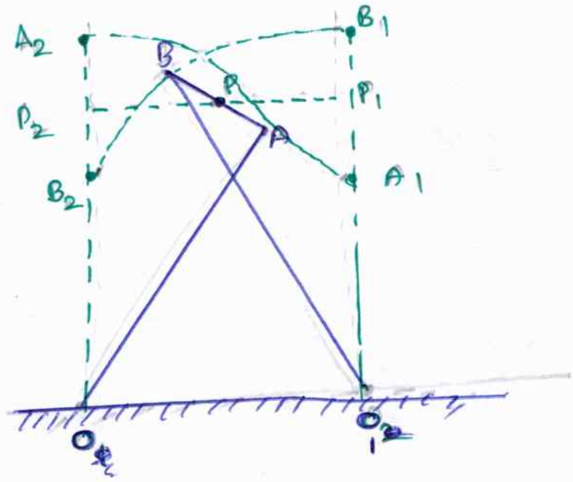
This mechanism is a modification of modified- Scott Russel's mechanism with the difference that the point 'p' does not slide along a straight line, but moves in a circular arc with centre 'O_1'

It is a four bar mechanism and all the pairs are turning pairs as shown in fig. In this mechanism, the centeres 'O' and 'O_1' are fixed. The link 'OA' oscillates about 'O' through an angle AOA, which causes the pin 'p' to move along a circular arc with 'O_1' as a center and O_1P as a radius. for small angular displacements of 'OP' on each side of the horizontal, the point 'Q' on the extension of the link 'PA' traces out an approximately a straight line path QQ^1.

If the lengths are such that $OA = \frac{(AP)^2}{AQ}$

Z

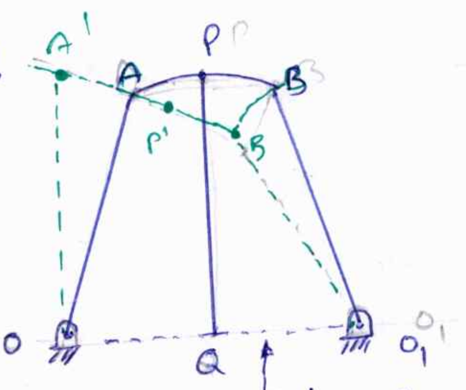
4. Tchebicheff's Mechanism



fig

It is a four bar mechanism in which the crossed links OA and O_1B are of equal lengths as shown in fig. The point 'P', which is the mid point of 'AB' traces out an approximately straight line parallel to OO_1. The proportions of the links are, usually, such that the point 'P' is exactly above 'O' or 'O_1' in the extreme positions of the mechanism i.e when 'BA' lies along 'OA' or when BA lies along O_1B. It may be noted that the point 'P' will be lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths are such that 'OA' = $(AP)^2/AQ$.

5. Roberts mechanism



It is also a four bar chain mechanism, which in its mean position, has the form of a trapezium. The link 'OA' and 'O_1B' are of equal length and OO_1 is fixed. A bar 'PQ' is rigidly attached to the link AB at its middle point 'P'. A little consideration will show that if the mechanism is displaced as shown by the dotted lines in fig. The point 'Q' will trace out an approximately straight line.

UNIT - III

STEERING Mechanisms

The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path. Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.

In automobiles, the front wheels are placed over the front axles, which are pivoted at the points 'A' and 'B'. as shown in fig. These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight and do not turn. Therefore the steering is done by means of front wheels only

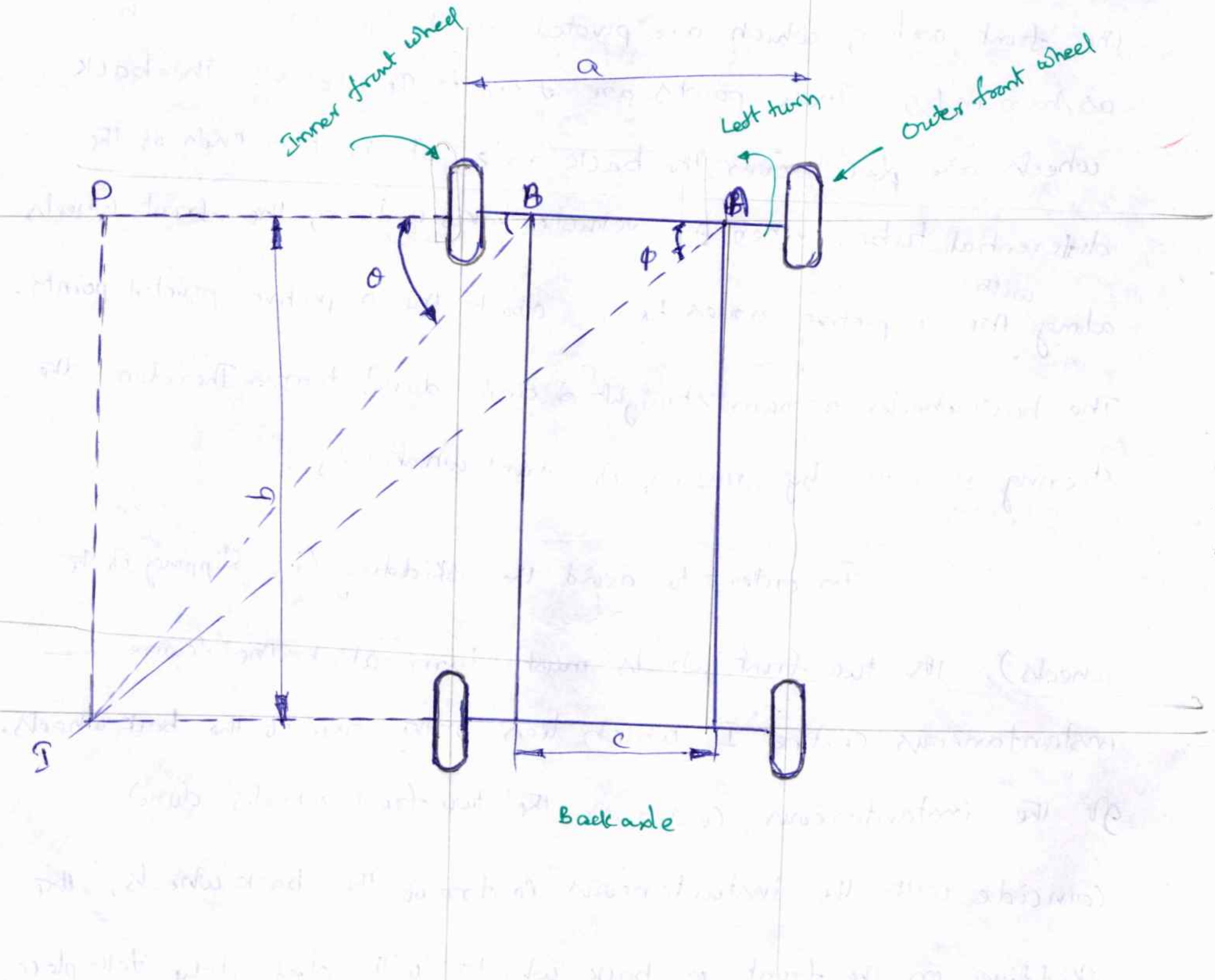
In order to avoid the skidding (i.e slipping of the wheels), the two front wheels must turn about the same instantaneous centre 'I' which lies on the axis of the back wheels. If the instantaneous center of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.

Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheels makes a larger turning angle θ than the angle ϕ subtended by the axis of the outer wheel.

Let a = wheel track

b = wheel base, and

c = Distance between the pivots A and B of the front axle.



Now from Triangle IPB

$$\cot\theta = \frac{BP}{IP}$$

from Triangle IPA

$$\cot\phi = \frac{AP}{IP} = \frac{BP + AB}{IP} = \frac{BP}{IP} + \frac{AB}{IP}$$

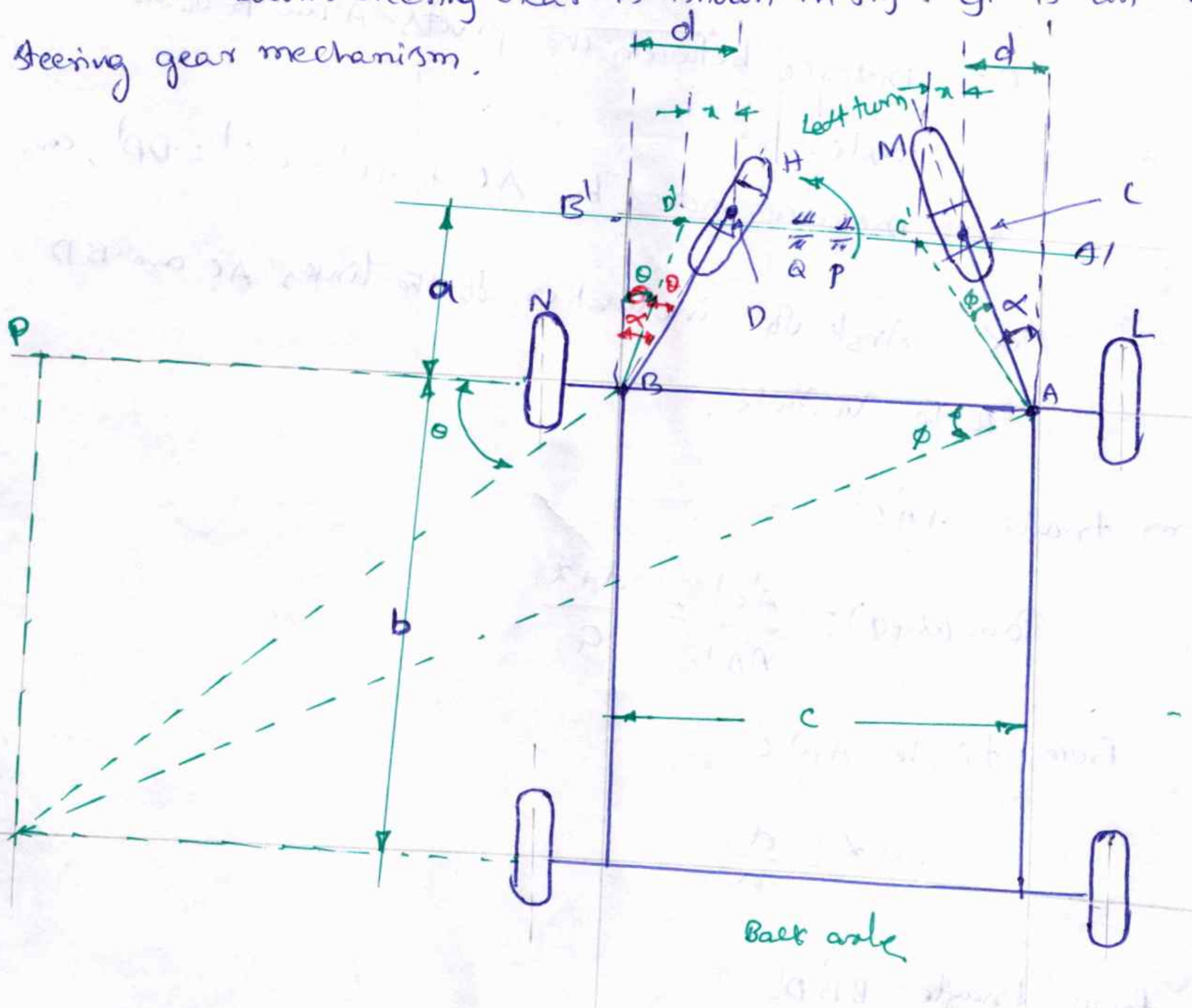
$$\Rightarrow \quad \cot\phi = \cot\theta + \frac{c}{b}$$

$$\Rightarrow \quad \boxed{\cot\phi - \cot\theta = \frac{c}{b}}$$

The above equation is the fundamental equation for correct equation.

DAVIS STEERING GEAR

The Davis Steering Gear is shown in fig. It is an exact steering gear mechanism.



The slotted links AM and BH are attached to the front wheel axle, which turn on pivots 'A' and 'B' respectively. The rod 'CD' is constrained constrained to move in the direction of its length, by the sliding members at 'P' and 'Q'. These constraints are connected to the slotted link 'AM' and 'BH' by a sliding and turning pair at each end. The steering is affected by moving 'CD' to the right or left of its normal position. C'D' shows the position of 'CD' for turning to the left.

Let a = Vertical distance between AB and CD

b = Wheel base

d = horizontal distance between the 'AC' and 'BD'.

c = Distance between the pivots 'A' and 'B' of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD to the vertical.

From triangle $AA'C'$

$$\tan(\alpha+\phi) = \frac{A'C'}{AA'} = \frac{d+x}{a}$$

from triangle $AA'C$

$$\tan\alpha = \frac{d}{a}$$

from triangle $BB'D'$

$$\tan(\alpha-\phi) = \frac{d-x}{a}$$

we know that $\tan(\alpha+\phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha \times \tan\phi}$

$$\Rightarrow \frac{d+x}{a} = \frac{\frac{d}{a} + \tan\phi}{1 - \frac{d}{a} \times \tan\phi}$$

$$\Rightarrow \frac{d+x}{a} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

$$\Rightarrow (d+x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$\Rightarrow \cancel{ad} - d^2\tan\phi + xa - xd\tan\phi = \cancel{ad} + a^2\tan\phi$$

$$\Rightarrow a^2\tan\phi + d^2\tan\phi + xd\tan\phi = xa$$

$$\Rightarrow (a^2 + d^2 + xd) \times \tan\phi = xa$$

$$\Rightarrow \tan\phi = \frac{xa}{(a^2 + d^2 + xd)}$$

ly from $\tan(\alpha - \phi) = \dfrac{\tan\alpha - \tan\phi}{1 + \tan\alpha \tan\phi}$

$$\Rightarrow \tan\theta = \frac{xa}{a^2 + d^2 - xd}$$

$\therefore$ from the condition of correct steering

$$\cot\phi - \cot\theta = \frac{c}{b} \Rightarrow \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\Rightarrow \frac{a^2 + d^2 + xd}{xa} - \frac{a^2 + d^2 - xd}{xa} = \frac{c}{b}$$

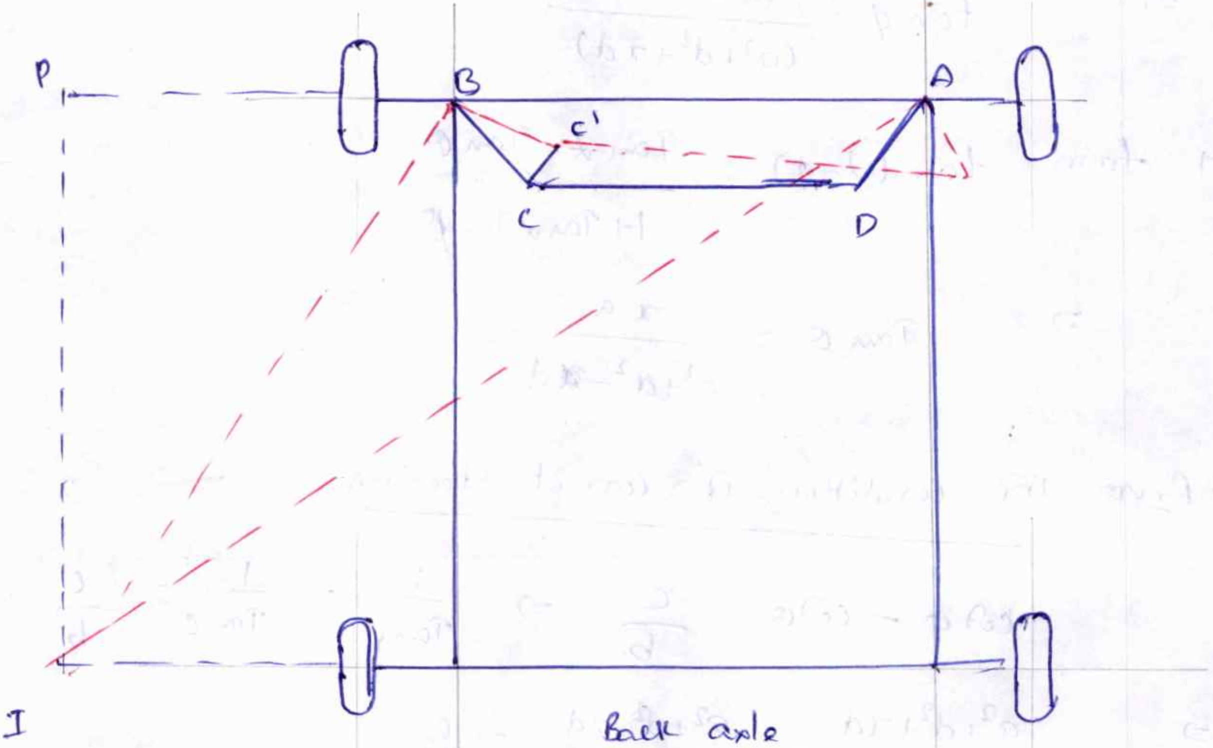
$$\Rightarrow \frac{(a^2 + d^2 + xd) - (a^2 + d^2 - xd)}{xa} = \frac{c}{b}$$

$$\Rightarrow \frac{2xd}{xa} = \frac{c}{b} \Rightarrow \boxed{\frac{d}{a} = \frac{c}{2b}}$$

$$\therefore \boxed{\tan\alpha = \frac{c}{2b}}$$

Note:- Though the gear is theoritically correct, but due to the presence of more sliding members, the wear will be increased which produces slackness between the sliding surfaces, thus eliminating the original accuracy. Hence Davis steering gear is not in common use.

Ackerman Steering Gear



The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are:

1. The whole mechanism of the Ackerman steering gear is on the back of the front wheels. where as in Davis steering gear, it is infront of the wheels.

2. The Ackerman steering gear consists of turning pairs, where as the davis steering gear consists of sliding members.

In Ackerman steering gear, the mechanism ABCD is a four bar crank chain as shown in fig. The shorter links BC and AD are of equal lengths and are connected by hinge joints with front wheel axles. The longer links AB and CD are of unequal length. The following are the only three positions for correct steering.

1. when the vehicle moves along a straight path, the longer links AB and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by Thick (firm) lines in fig.

2. When the vehicle is steering to the left, The position of the gear is shown by dotted lines in fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.

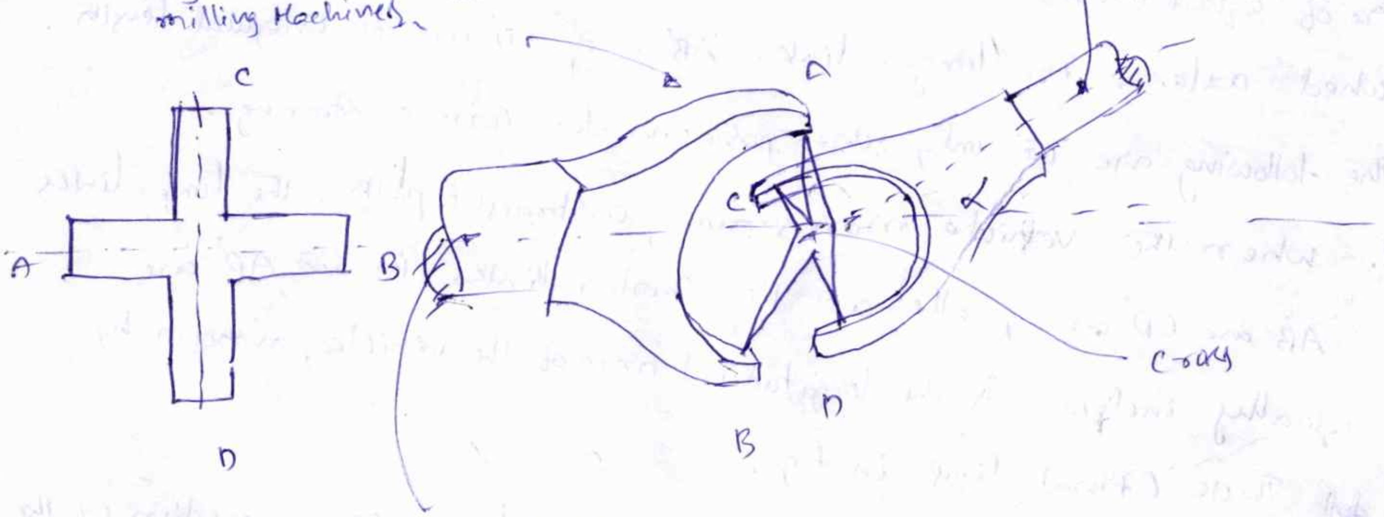
3. when the vehicle is steering to the right, the similar position may be obtained.

In order to satisfy the fundamental equation for correct steering the links AD and DC are suitably proportioned.

Universal (or) Hooke's joint

A Hooke's joint is used to connect two shaft, which are intersecting at a small angle, as shown in fig. The end of each shaft is forked to U-type and each fork is provided with two bearings for the arms of a cross. The arms of the cross is perpendicular to each other. The motion is transmitted from the driving shaft to the driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted.

Application:- (1) Transmission from GearBox to Differential (Automobile) [Double Hook joint is used.]
(2) Transm of power to different spindles of multiple drilling Machine
(3) Knee joint in milling Machines.

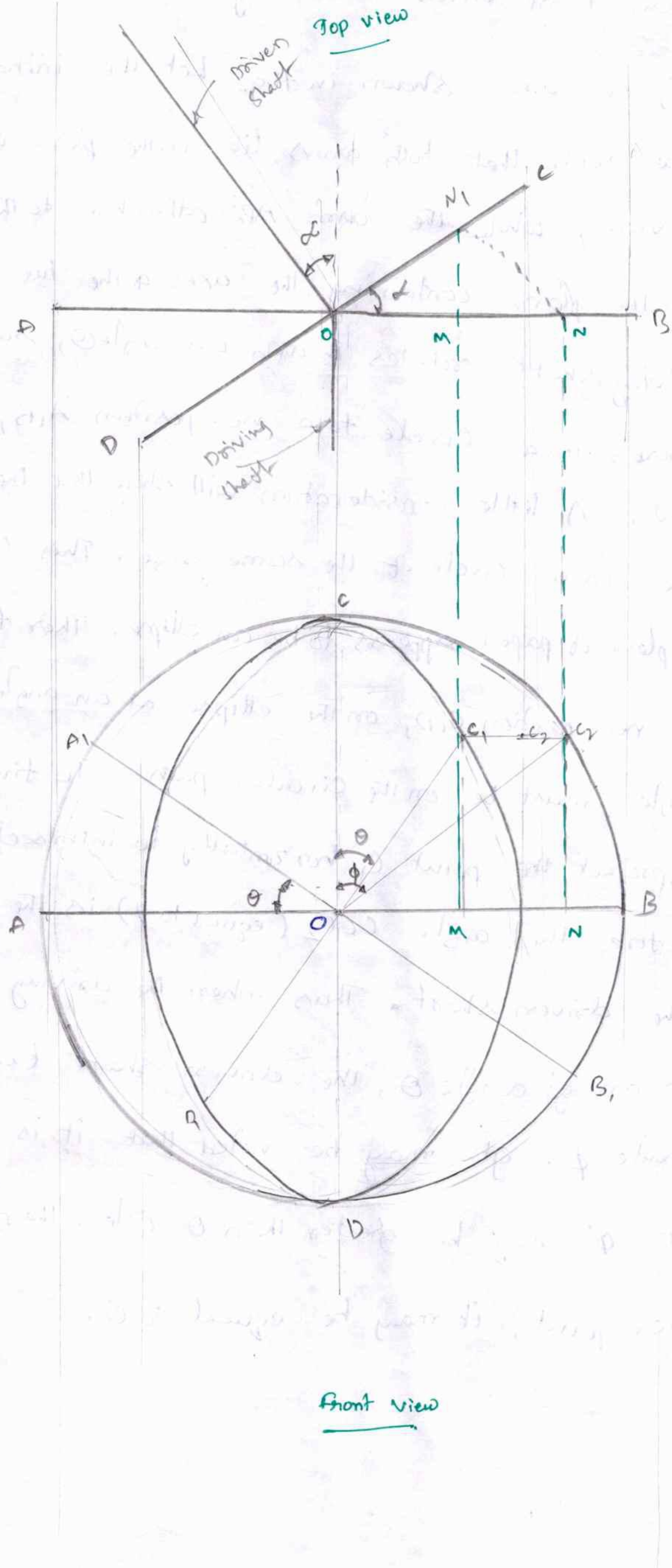


(P). In a Davis steering gear, the distance between the pivots of the front axle is 1.2m and the wheel base is 2.7m. Find the inclination of the track arms to the longitudinal axis of the car, when it is moving along a straight path.

(Sol: $\tan\alpha = \frac{c}{2b} = \frac{1.2}{2\times 2.7} = 0.222$; $\alpha = 12.5^\circ$)

Ratio of the Shaft Velocities

2,11,12, 15,12



The top and front views connecting the two shafts by a Universal joint are shown in fig. Let the initial position of the cross be such that both arms lie in the plane of paper in the front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shaft. Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A_1B_1 as shown in front view. A little consideration will show that the arm CD will also move in a circle of the same size. This circle, when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C_1D_1 on the ellipse at an angle θ. But the true angle must be on the circular path. To find the true angle, project the point C_1 horizontally to intersect the circle at C_2. Therefore the angle COC_2 (equal to ϕ) is the true angle turned by the driven shaft. Thus when the driving shaft turns through an ~~of~~ angle θ, the driven shaft turns through an angle ϕ. It may be noted that it is not necessary that 'ϕ' may be greater than 'θ' or less than θ. At a particular point, it may be equal to θ.

from $\angle OC_1M \Rightarrow$ $\tan\theta = \frac{OM}{C_1M}$ — (i)

from $\angle OC_2N \Rightarrow$ $\tan\phi = \frac{ON}{C_2N}$ — (ii)

from topview $ON = ON_1 \Rightarrow$ $|||^{ly}$ $OM = ON_1\cos\alpha = ON\cos\alpha$

$\Rightarrow \frac{(i)}{(ii)} \Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{\frac{OM}{C_1M}}{\frac{ON}{C_2N}} = \frac{OM}{ON}$ $[\because C_1M = C_2N]$

$\Rightarrow \frac{\tan\theta}{\tan\phi} = \frac{\cancel{ON}\cos\alpha}{\cancel{ON}}$

$\Rightarrow \boxed{\tan\theta = \tan\phi\cos\alpha}$

Differentiating the above equation w.r.t. t both sides

$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\phi \times \frac{d\phi}{dt}$ — (A)

Let assume θ is the angular displacement of driving shaft
ω = Angular speed of driving shaft $= \frac{d\theta}{dt}$

$|||^{ly}$ ϕ is the angular displacement of driven shaft
ω_1 = Angular speed of driven shaft $= \frac{d\phi}{dt}$

$\sec^2\theta \times \omega = \cos\alpha \times \sec^2\phi \cdot \omega_1$

$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\phi}$ — (B)

from fundamentals

$\sec^2\phi = 1 + \tan^2\phi = 1 + \left(\frac{\tan\theta}{\cos\alpha}\right)^2 = 1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}$

$\Rightarrow \sec^2\phi = \frac{\cos^2\theta \cdot \cos^2\alpha + \sin^2\theta}{\cos^2\theta\cos^2\alpha} = \frac{(1 - \sin^2\alpha)\cos^2\theta + \sin^2\theta}{\cos^2\theta\cos^2\alpha}$

$\Rightarrow \sec^2\phi = \frac{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}{\cos^2\theta\cos^2\alpha} = \frac{1 - \cos^2\theta\sin^2\alpha}{\cos^2\theta\cos^2\alpha}.$

$\Rightarrow$

$$\Rightarrow \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \sec^2\phi} = \frac{\cos^2\theta \cos^2\alpha}{\cos^2\theta \times \cos\alpha\,(1-\cos^2\theta\sin^2\alpha)}$$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}}$$

(or)

$$\boxed{\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}}$$

where N_1 = Speed of driving shaft in r.p.m

N = Speed of driven shaft in r.p.m

Maximum and Minimum Speeds of Driven Shaft

from the Ratio of shaft speeds.

$$\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

N_1 is maximum when $(1-\cos^2\theta\sin^2\alpha)$ is minimum

it is possible when $\theta = 0°$ or $180°$ or $360°$.

$$\therefore \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2 0°\sin^2\alpha} = \frac{\cos\alpha}{1-\sin^2\alpha} = \frac{\cos\alpha}{\cos^2\alpha}$$

$$\Rightarrow \frac{N_1}{N} = \frac{1}{\cos\alpha}$$

$$\Rightarrow \boxed{N_{1\,(max)} = \frac{N}{\cos\alpha}}$$

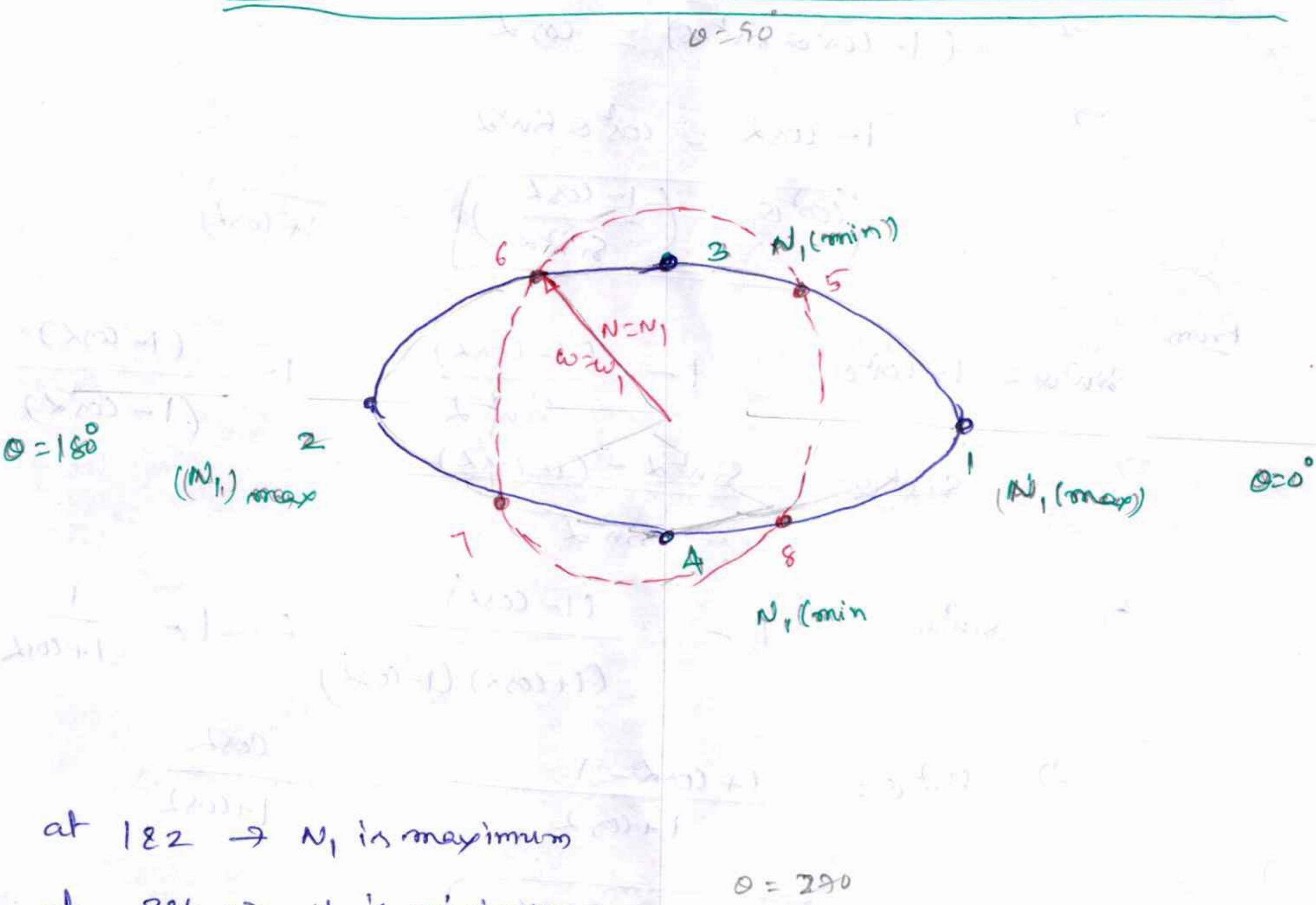
$\|^{ly}$ N_1 is minimum when $(1-\cos^2\theta \sin^2\alpha)$ is minimum

it is possible when $\theta = 90°, 270°$ etc.

$$\therefore \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2 90 \sin^2\alpha} = \frac{\cos\alpha}{1-0\times\sin^2\alpha} = \cos\alpha$$

$$\therefore \boxed{N_{1\,(min)} = N\cos\alpha}$$

Polar Diagram - Salient features of Driven shaft speed



at 1 & 2 → N_1 is maximum

at 3 & 4 → N_1 is minimum

at 5, 6, 7, 8 → N_1 is equal to N

Condition for equal Speeds of the Driving & Driven Shaft

From Ratio of Speeds of Shafts

$$\frac{\omega_1}{\omega} = \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

If the Speed of Driving and Driven Shaft is same

$N_1 = N \Rightarrow 1 = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$

$\Rightarrow (1-\cos^2\theta\sin^2\alpha) = \cos\alpha$

$\Rightarrow 1-\cos\alpha = \cos^2\theta\sin^2\alpha$

$\Rightarrow \boxed{\cos^2\theta = \left(\frac{1-\cos\alpha}{\sin^2\alpha}\right)} = \frac{1}{(1+\cos\alpha)}$

From

$\sin^2\theta = 1-\cos^2\theta = 1-\frac{(1-\cos\alpha)}{\sin^2\alpha} = 1-\frac{(1-\cos\alpha)}{(1-\cos^2\alpha)}$

$\Rightarrow \sin^2\theta = \frac{\sin^2\alpha-(1-\cos\alpha)}{\sin^2\alpha} =$

$\Rightarrow \sin^2\theta = 1-\frac{(1-\cos\alpha)}{(1+\cos\alpha)\cancel{(1-\cos\alpha)}} = 1-\frac{1}{1+\cos\alpha}$

$\Rightarrow \sin^2\theta = \frac{1+\cos\alpha-1}{1+\cos\alpha} = \frac{\cos\alpha}{1+\cos\alpha}$

$$\therefore \boxed{\sin^2\theta = \frac{\cos\alpha}{1+\cos\alpha}}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{\frac{\cos\alpha}{1+\cos\alpha}}{\frac{(1-\cos\alpha)}{\sin^2\alpha}} = \frac{\cos\alpha}{(1+\cos\alpha)}\times\frac{\sin^2\alpha}{(1-\cos\alpha)}$$

$$\Rightarrow \tan^2\theta = \frac{\cos\alpha \times \sin^2\alpha}{1-\cos^2\alpha} = \frac{\cos\alpha \times \cancel{\sin^2\alpha}}{\cancel{\sin^2\alpha}}$$

$$\Rightarrow \tan^2\theta = \cos\alpha$$

$$\Rightarrow \boxed{\tan\theta = \pm\sqrt{\cos\alpha}}$$

There are two values of 'θ' corresponding to positive sign and two values corresponding to negative sign. Hence, there are four values of 'θ', at which the speeds of the driving and driven shafts are same. This is shown by points 5, 6, 7 and 8 in polar diagram.

Maximum fluctuation of speed:

We know that the maximum speed of the driven shaft

$$\boxed{N_{1(max)} = \frac{N}{\cos\alpha}}$$

and minimum speed of the driven shaft

$$\boxed{N_{1(min)} = N\cos\alpha}$$

The maximum fluctuating speed of driven shaft (q) is equal to the difference between the maximum and minimum speeds of the driven shaft.

$$\therefore q = N_{1(max)} - N_{1(min)} = \frac{N}{\cos\alpha} - N\cos\alpha$$

$$\Rightarrow q = N\left(\frac{1}{\cos\alpha} - \cos\alpha\right)$$

$$\Rightarrow q = N_{\beta} \frac{1-\cos^2\alpha}{\cos\alpha} = N_{\beta} \frac{\sin^2\alpha}{\cos\alpha}$$

$$\Rightarrow \boxed{q = N \tan\alpha \times \sin\alpha}$$

(or)

$$\boxed{q = \omega \tan\alpha \sin\alpha}$$

Since 'α' is a small angle therefor substitution $\cos\alpha = 1$ and $\sin\alpha = $ 'α' Radians

$$\Rightarrow q = \omega \times \frac{\sin\alpha}{\cos\alpha} \times \sin\alpha = \omega \frac{\alpha \times \alpha}{1}$$

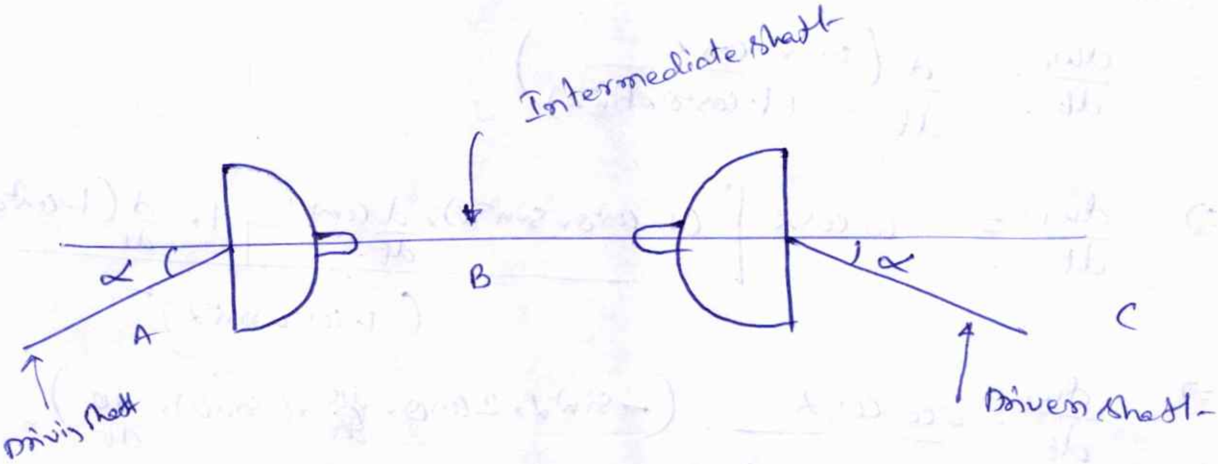
$$\Rightarrow \boxed{q = \omega \times \alpha^2 \text{ or } N\alpha^2}$$

$\therefore$ Maximum fluctuation Speed $\boxed{q_{(max)} = \omega\alpha^2}$

Hence, the maximum fluctuation of speed of the driven shaft approximately varies as the square of the angle between the two shaft.

Double Hooke's joint

We know that the velocity of driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in fig. is used. This type of joint is known as double Hooke's joint.



Let the driving, intermediate and driven shafts in the same time, rotate through an angles θ, ϕ and γ from the position

Now for shafts A and B $\quad \tan\theta = \tan\phi \cos\alpha \quad$ —(i)

for shafts B and C $\quad \tan\gamma = \tan\phi \cos\alpha \quad$ —(ii)

from equations (i) and (ii) we see that $\theta = \gamma$; $N_A = N_C$

This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if

1. The axes of the driving and driven shafts are in the same plane
2. The driving and driven shaft makes equal angles with the intermediate shaft.

Angular Acceleration of the Driven Shaft

From $\frac{\omega_1}{\omega} = \frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}$

$$\omega_1 = \omega \frac{(\cos\alpha)}{(1-\cos^2\theta \sin^2\alpha)}$$

Differentiating the above expression, w.r.t. t ; we obtain the angular acceleration.

$$\frac{d\omega_1}{dt} = \frac{d}{dt}\left(\omega \times \frac{\cos\alpha}{(1-\cos^2\theta \times \sin^2\alpha)}\right)$$

$$\Rightarrow \frac{d\omega_1}{dt} = \omega\cos\alpha \left[\frac{(1-\cos^2\theta \times \sin^2\alpha) \times \frac{d}{dt}(\text{const}) - 1 \times \frac{d}{dt}(1-\cos^2\theta \times \sin^2\alpha)}{(1-\cos^2\theta \sin^2\alpha)^2}\right]$$

$$\Rightarrow \frac{d\omega_1}{dt} = \frac{-\omega\cos\alpha}{(1-\cos^2\theta \sin^2\alpha)^2}\left(-\sin^2\alpha \times 2\cos\theta \times \cancel{\frac{d\theta}{dt}} \times (-\sin\theta) \times \frac{d\theta}{dt}\right)$$

$$\Rightarrow \boxed{\frac{d\omega_1}{dt} = \frac{-\omega^2 \cos\alpha(\sin 2\theta \times \sin^2\alpha \times \cancel{\omega})}{(1-\cos^2\theta \sin^2\alpha)^2}}$$

[∴ -ve indicates the Retardation]

For angular acceleration to be maximum, differentiate $\frac{d\omega_1}{dt}$ with respect to θ and equate to zero. The result is approximated as

$$\boxed{\cos 2\theta = \frac{\sin^2\alpha\,(2-\cos^2 2\theta)}{2-\sin^2\alpha}}$$

Note:- If the value of 'α' is less than 30°, then $\cos 2\theta$ may approximately be written as

$$\boxed{\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}}$$

Condition for Max Angular accln

(P) Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required.

$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}$; $\theta = ✓$

$(T_{max} = I\alpha_{(max)})$ $= I\frac{d\omega}{dt}$; $I = mk^2$

(Ans:- $\theta = 41.65°$, $\frac{d\omega_1}{dt} = 3090$ rad/s² (Max) ; $T = 371$ N-m ; $I = 0.12$ kg-m²)

put a value in → ; = max (+ve take)

(P) The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and unity.

(Ans: $\theta = 0°, 180°$; $\theta = 44.3°$ or $135.7°$) ($\cos\theta = \pm 0.7159$)

(i) $\left(\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1-\cos^2\theta \sin^2\alpha}\right.$; (ii) : $1 = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$

(P) Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500 r.p.m. If the total permissible variation in speed of driven shaft is not to exceed ±6% of the mean (ω) speed, find the greatest permissible angle between the centre lines of the shafts. (Ans $\alpha = 19.64°$)

$q = 0.12\omega$ ✓ $q = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$; $\cos\alpha = 0.94$; $\alpha = 19.64°$; $\left(= \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right.$

(P) Two shafts are connected by a Universal joint. The driving shaft rotates at a uniform speed of 1200 rpm. Determine the greatest permissible angle between the shaft axes so that the total fluctuation of speed does not exceed 100 r.p.m. Also calculate the maximum and minimum speeds of the driven shaft.

(Ans $\alpha = 16.4°$; $N_{1(max)} = 1251$ r.p.m ; $N_{1(min)} = 1157$ r.p.m)

$q = N\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right)$

$q = 100$; $N = 1200$; $\alpha = 16.4°$

$N_{1(max)} = \frac{N}{\cos\alpha}$; $N_{1(min)} = N\cos\alpha$

(P) The driving shaft of a Hooke's joint runs at a uniform speed of 240 r.p.m and the angle 'α' between the shafts is 20°. The driven shaft with attached masses has a mass of 55 kg at a radius of gyration of 150 mm.

1. If a steady torque of 200 N-m resists rotation of the driven shaft, find the torque required at the driving shaft when $\theta = 45^\circ$.

2. At what value of 'α' will the total fluctuation of speed of the driven shaft be limited to 24 r.p.m.

(Ans $T' = 102.6$ N-m ; $\alpha = 18.2^\circ$)

(P) A Double Universal joint is used to connect two shafts in the same plane. The intermediate shaft is inclined at an angle of 20° to the driving shaft as well as the driven shaft. Find the maximum and minimum speed of the intermediate shaft and the driven shaft if the driving shaft has a constant speed of 500 r.p.m. ($N_{B(max)} = 532.1$ rpm, $N_{B(min)} = 469.85$ rpm)

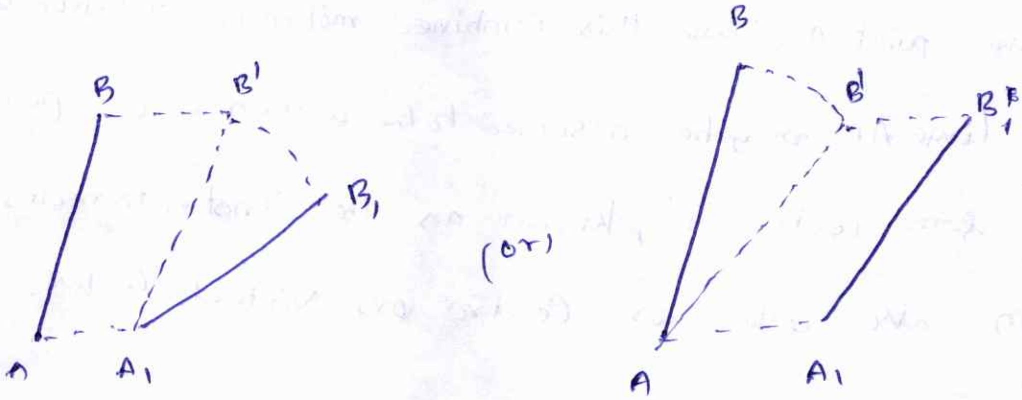
$N_{C(max)} = 566.25$ r.p.m.

$N_{C(min)} = 441.5$ rpm.

UNIT-IV (Kinematics, Analysis of Mechanisms and plane motion of Body)

Plane Motion of Body

Sometimes, a body has simultaneously a motion of rotation as well as translation, such as wheel of a car, a sphere rolling on a ground (without slipping). Such a motion will have the combined effect of rotation and translation.



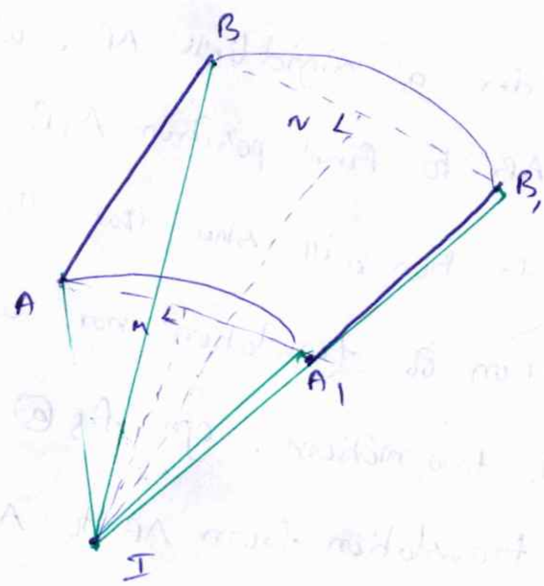
(a) (b)

motion of link

Consider a rigid link AB, which moves from its initial position AB to final position A_1B_1 as shown in fig. (a). A little consideration will show that the link neither has wholly a motion of translation nor wholly rotational, but a combination of two motions. In fig (a), the link has first the motion of translation from AB to A_1B' and then the motion of rotation about A_1, till it occupies the final position A_1B_1. Similarly in fig (b), the link AB has first the motion of rotation from AB to A_1B' about A and then the motion of translation from AB' to A_1B_1. Such a motion of link AB to A_1B_1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first or the motion of translation.

In actual practice, the motion of link AB is so gradual that it is difficult to see the two separate motions. But we see the two seperate motions, though the point B moves faster than the point A. Thus this combined motion of rotation and translation of the link AB may be assumed to be a motion of pure rotation about some center 'I', known as the Instantaneous Center of rotation also called as Centro (or) Virtual Center. The

How to Locate an Instantaneous Center



Since the points A and B of the link has moved to A_1 and B_1 respectively under the motion of rotation, therefore the position of center of rotation must lie on the intersection of the right bisectors of chords AA_1 and BB_1. Let these bisector intersect at I as shown in fig, which is the instantaneous center of rotation or virtual center of the link AB.

From the above we see that the position of the link AB goes on changing, therefore the center about which the motion is assumed to take place (i.e, instantaneous centre of rotation) also goes on changing. Thus the instantaneous center of moving body may be defined as that centre which goes on changing from one instant to another. The locus of all such instantaneous centers is known as Centrode. A line drawn through an instantaneous center and perpendicular to the plane of motion is called instantaneous axis. The locus of this axis is known as axode.

Space & Body Centrode

A Rigid body in plane motion relative to second rigid body, supposed fixed in space, may be assumed to be rotating about an instantaneous centre at that particular moment. In other words, the instantaneous center is a point in the body which may be considered fixed at any particular moment. "The locus of the instantaneous center in space during a definite motion of the body is called the 'SPACE CENTRODE' and the locus of the instantaneous centre relative to the body itself is called the 'BODY CENTRODE'. These two centrodes have the instantaneous center as a common point at any instant and during the motion of the body, the body centrode rolls without slipping over the space centrod.

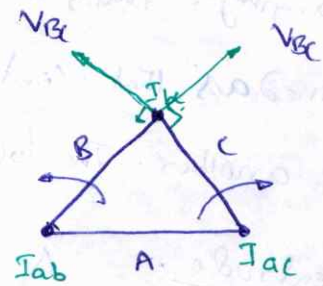
Aronhold Kennedy (or Three Centres in Line) Theorem

'The Aronhold Kennedy's theorem states that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.'

Consider three kinematic links A, B and C having relative plane motion. The Number of instantaneous Centers (N) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

where n = number of links = 3



The two instantaneous centres at the pin joints of B with A, and C with A are (I_{ab} & I_{ac}) the permanent instantaneous Centers, According to the Kennedy's Theorem the third instantaneous Centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us Consider that the instantaneous Centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in fig. The point I_{bc} b

Properties of the Instantaneous Centre

1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous center. At this point, the two rigid links have the same linear velocity relative to the third rigid link.

Numbers of Instantaneous centers in a Mechanism

The number of instantaneous centers in a constrained kinematic chain is equal to the number of possible combinations of two links. Mathematically the number of instantaneous centers (N)

$\Rightarrow \quad N = \dfrac{n(n-1)}{2}$, where n = number of links.

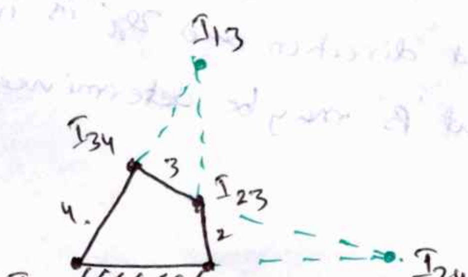
Types of instantaneous centers

The instantaneous centers for a mechanism are of the following 3 types

1. Fixed instantaneous centers
2. Permanent Instantaneous centers

} Primary Instantaneous Center

3. Neither fixed nor Permanent instantaneous center → Secondary Instantaneous Centers.



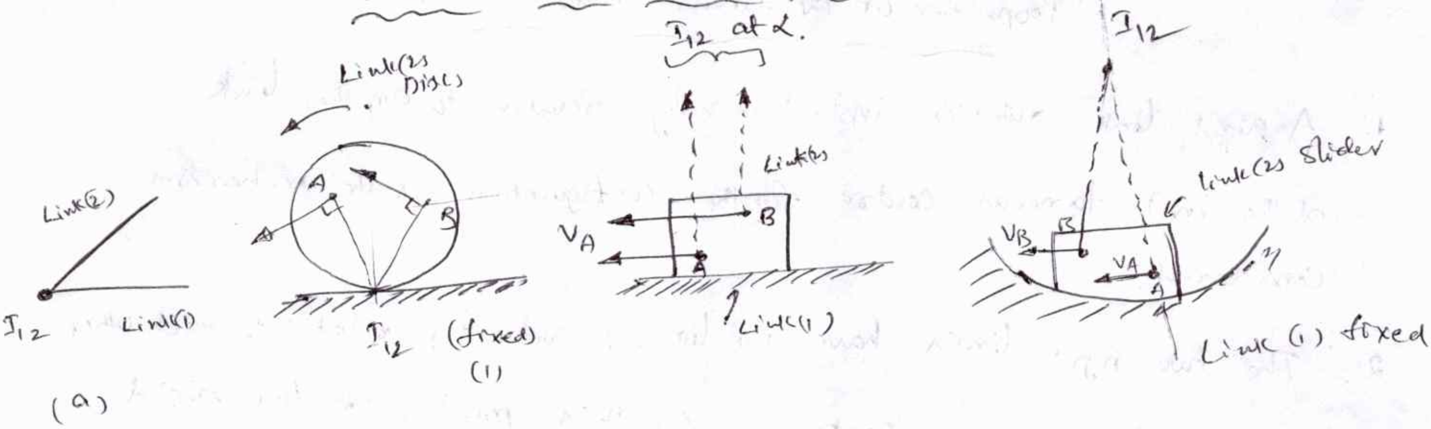
I_{12}	I_{23}	I_{34}
I_{13}	I_{24}	
I_{14}		

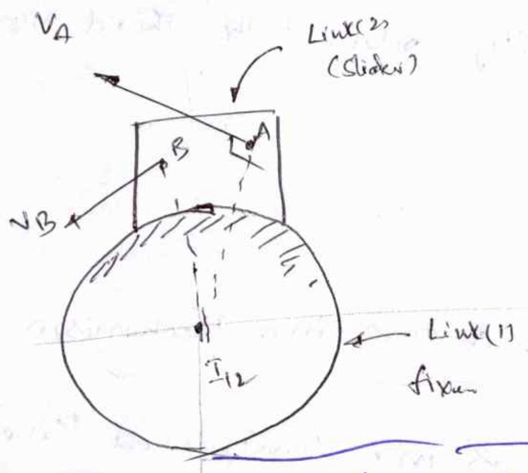
I_{12} & I_{14} → fixed

I_{34} & I_{23} → Permanent

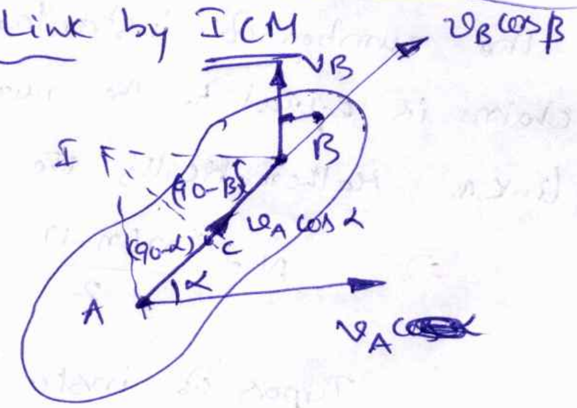
I_{13}, I_{24} → neither fixed nor Permanent

Location of Instantaneous Centres





Velocity of a Point on a Link by ICM



$$v_A \cos\alpha = v_B \cos\beta$$

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(9-\alpha)}$$

(ABI) from Lami's theorem

$$\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}$$

$$\Rightarrow \frac{\sin(90-\beta)}{\sin(90-\alpha)} = \frac{AI}{BI}$$

$$\therefore \frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{(or)} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} = \omega$$

From the above:-

1. If 'v_A' is known in magnitude and direction and 'v_B' is in direction only, then the velocity of a point 'B' may be determined in magnitude and direction.

Method of Locating Instantaneous Centres in a Mechanism

(1) First of all, Determine the number of instantaneous centres (N) by using the relation

$$N = \frac{n(n-1)}{2}, \quad \text{where } n = \text{Number of links}$$

(2) Mark the list of all instantaneous centers in a mechanism.

Draw the table chart for this

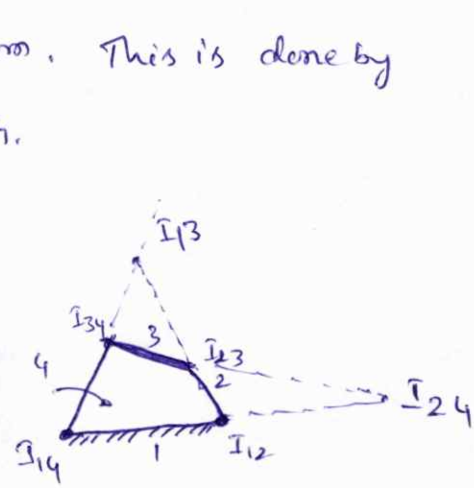
Ex:- 4 links → $N = 6$ ($\because \frac{4(4-1)}{2} = 6$) (Four bar chain)

Number of links	1	2	3	4	. . .
No. of Instantaneous Centers	I_{12}	I_{23}	I_{34}	–	
	I_{13}	I_{24}	–	–	
	I_{14}	–	–	.	

(3) Locate the fixed and permanent Instantaneous Centres by inspection.

(4) Locate the remaining neither fixed nor permanent instantaneous centres (Secondary Centres) by Kennedy's theorem. This is done by circle diagram as shown in example fig.

Ex:- Four bar chain



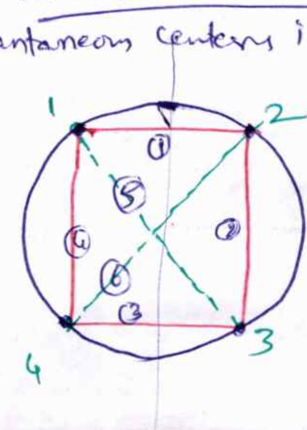
Four bar chain Mechanism

(5) Draw Circle and do the steps 3 & 4. i.e

(i) mark the points on a circle equal to the number of links in a mechanism

Ex:- (4) in this example. and locate the primary Instantaneous centers i.e $I_{14}, I_{12}, I_{23}, I_{34}$.

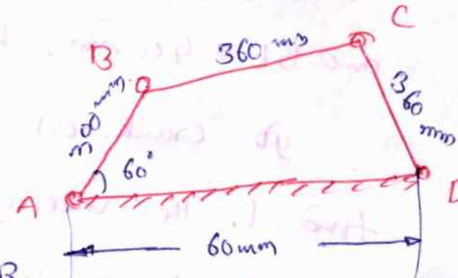
(ii) Locate the Secondary ~~for~~ instantaneous centers I_{13} & I_{24}



(P) In a pin joined four bar chain mechanism as shown in fig.

AB = 300mm, BC = CD = 360mm and AD = 600mm.

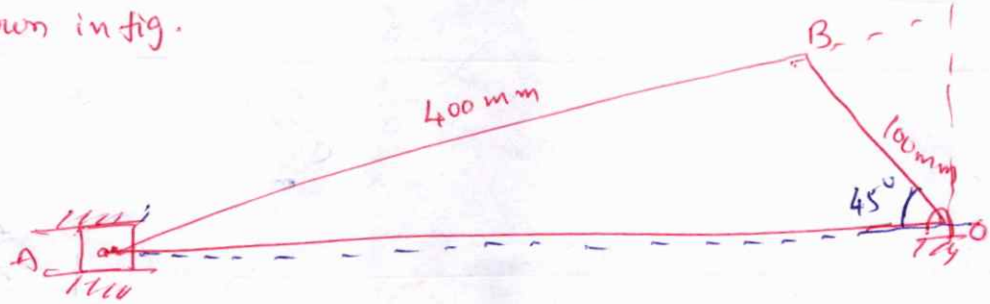
The angle BAD = 60°. The Crank AB rotates uniformly at 100 r.p.m. Locate the all instantaneous centers and find the angular velocity of the link BC.



$I_{13}B = 500mm = 0.5m$

(Ans:- $v_B = \omega_{AB} \times AB = \omega_{BC} \cdot I_{13} \times B = 6.282$ rad/sec)

(P) Locate the all instantaneous Centres of the slider crank mechanism as shown in fig.



~~Note~~ The lengths of crank OB and connecting rod AB are 100mm and 400mm respectively. If the crank rotates clock wise with an angular velocity of 10 rad/sec. find 1. Velocity of the slider A, and 2. Angular Velocity of the connecting rod AB.

(Ans. $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} \Rightarrow v_A = 0.82$ m/sec)

$\omega_{AB} = \frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = 1.78$ rad/sec.)

(D) A mechanism as shown in fig has the following dimensions.

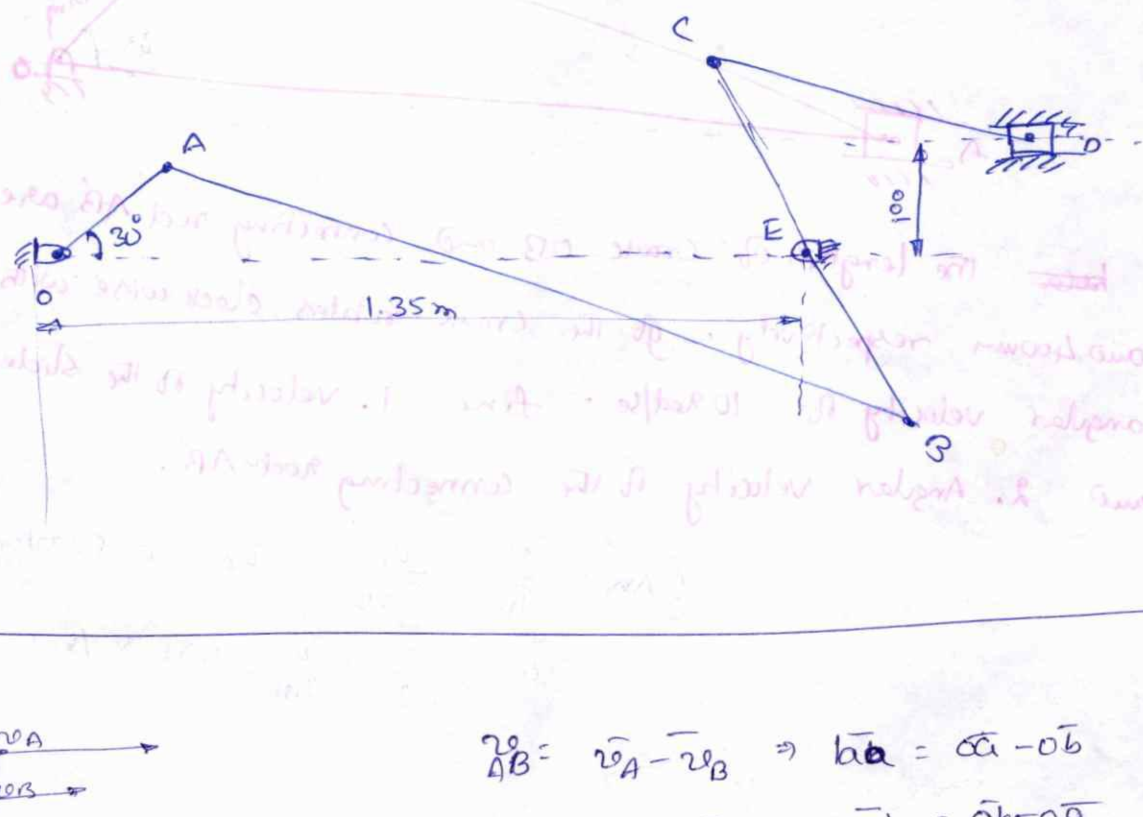
OA = 200mm ; AB = 1.5m = 1500mm ; BC = 600 mm ; CD = 500mm and BE = 400mm. Locate the all instantaneous centers.

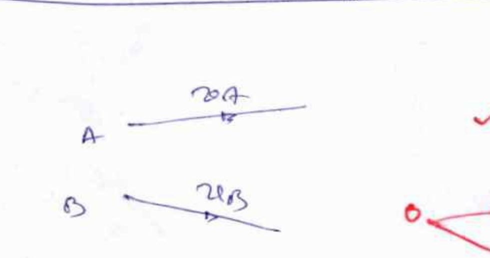
If crank OA rotates uniformly at 120 r.p.m. clockwise, find 1. the velocity of B, C and D

2. the angular velocity of the links AB, BC, and CD

(Ans:- $v_B = 3.2$ m/sec $v_C = 1.6$ m/sec $v_D = 1.05$ m/sec

$\omega_{AB} = 2.99$ rad/sec $\omega_{BC} = 8$ rad/sec $\omega_{CD} = 2.16$ rad/sec





$v_{AB} = \bar{v}_A - \bar{v}_B \Rightarrow \bar{ba} = \bar{oa} - \bar{ob}$

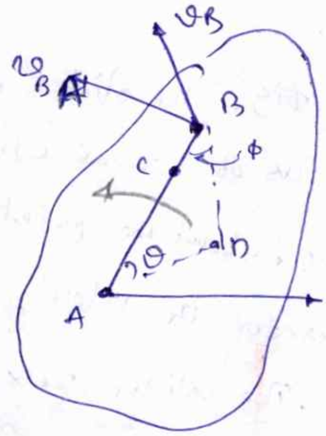
$v_{BA} = \bar{v}_B - \bar{v}_A \Rightarrow \bar{ab} = \bar{ob} - \bar{oa}$

Velocity in Mechanism

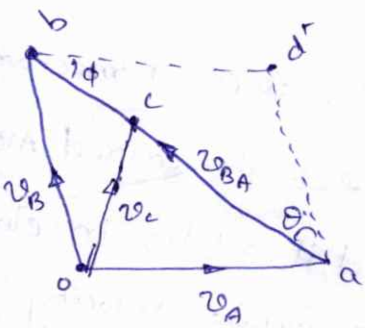
(1) Relative Velocity of Two Bodies moving in straight Lines

(2) Relative velocity of Two Bodies moving in an inclined path.

(3) Determining the velocity of a point on a Link by Relative velocity method.



Link

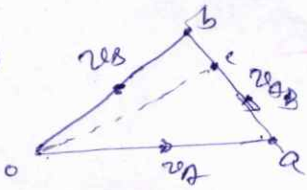


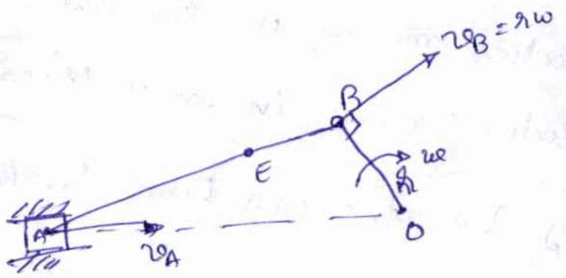
Velocity diagram.

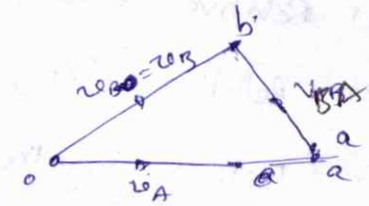
The relative velocity method is based on a link as shown in fig. Let the Absolute velocity of the point 'A' i.e v_A is known in magnitude and direction and the Absolute velocity of a point B i.e (v_B) is known in direction only. Then the velocity of 'B' is determined by drawing the velocity diagram. The procedure for drawing velocity diagram is as follows.

1. take a convenient point o, known as pole
2. Through 'o' draw 'oa' line parallel and equal to v_A, to some suitable scale
3. Through 'a', draw a line perpendicular to AB, this line represents the velocity of 'B' with respect to A i.e v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b.
5. Measure 'ob', which gives the required velocity of point B (v_B) to the scale.

Velocity in slider crank mechanism







(a) Slider crank Mechanism

velocity diagram

A slider ~~c~~ crank mechanism is shown in fig. the slider 'A' is attached to the Conrod AB. Let the radius of crank 'OB' be 'r' and let it rotates in clockwise direction, about the point 'O' with uniform angular velocity ω rad/sec. Therefor the velocity of B, is known in ~~mag~~ magnatude and direction. The slider reciprocates along the line of stroke AO. The velocity of slider may be determined by relative Velocity method. as discussed below.

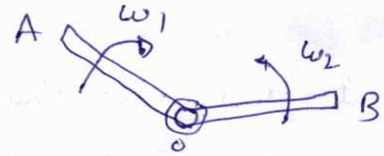
1. from any point 'o', draw vector ob' parallel to the direction of v_B (or perpendicular to OB) such that $ob = v_B = \omega r$, to some suitable scale.

2. Since AB is a rigid link, therefore the velocity of 'A' relative to 'B' is perpendicular to AB. Now draw the vector ba perpendicular to AB to represent the velocity of 'A' with respect to 'B', i.e v_{AB}.

3. From point 'o', draw vector 'oa' parallel to the path of motion of the slider A. The vectors ba and oa intersect at a, No 'oa' represents the velocity of slider 'A' ie v_A to scale.

$\therefore$ the angular velocity of con. Rod is determined as follows $\omega_{AB} = \dfrac{v_{BA}}{AB} = \dfrac{ab}{AB}$.

Rubbing velocity at a pin point



The Rubbing velocity at point O

$$v_0 = (\omega_1 \pm \omega_2) \times r$$

take +ve if the links move in opposite direction

take −ve if the links move in same direction

r = Radius of pin

The Rubbing velocity is defined as

or the algebraic sum of the angular velocity of two links which are connected by pin joints, ~~and~~ multiplied by the radius of pin.

---

(1) The fig. shows the structure of Whitworth Q.R. mechanism used in Reciprocating machine tools. The various dimensions of the tool are as follows; OQ = 100 mm; OP = 200 mm; RQ = 150 mm; RS = 500 mm. The crank makes an angle of 60° with the vertical. Determine the velocity of slider 'S' (cutting tool) when crank rotates at 120 rpm clockwise. Find also the angular velocity of link 'RS' and velocity of sliding block 'T' on slotted lever QT.

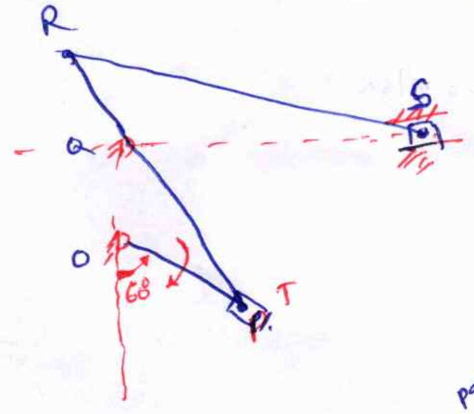
Sol: $N_{PO} = 120$ rpm; $\omega_{PO} = 12.57$ rad/s; $V_{PO} = V_P = 2.514$ m/sec

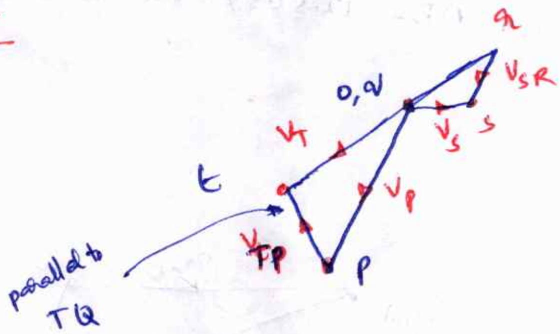
(Ans: $V_S \simeq 0.8$ m/s $= os$;

$V_{SR} = rs \simeq 0.96$ m/sec

$\omega_{RS} = 0.92$ rad/sec

$V_{tp} = pt \simeq 0.85$ m/sec





Problems

(1) In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when the angle BAD = 60°.

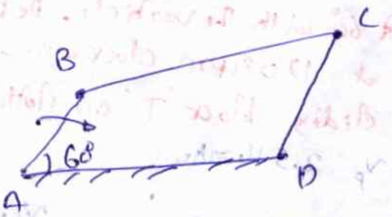
Sol:- Given Data:

$N_{AB} = 120\ rpm \Rightarrow \omega_{AB} = 12.568\ rad/s$,

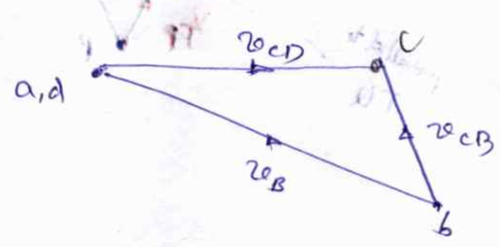
$AB = 40mm = 0.04m$

$BC = AD = 150\ mm$, $CD = 80mm = 0.08m$.

$\angle BAD = 60°$



$v_B = \omega_{AB} \cdot AB = 12.568 \times 0.04 = 0.503\ m/sec$



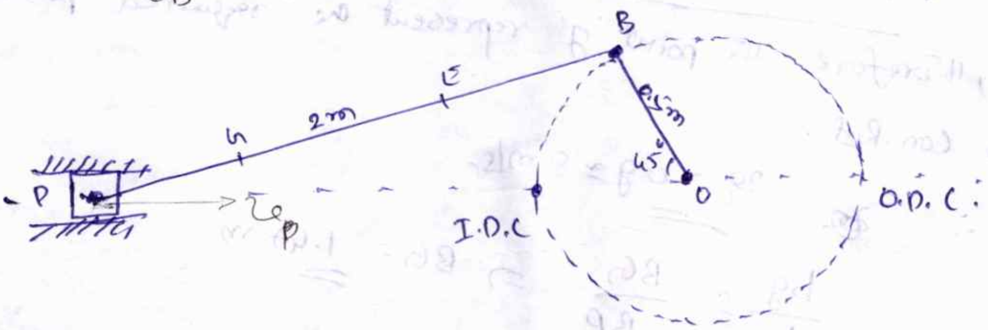
Calculate $v_{CD} = v_c =$ vector dc $= 0.385\ m/sec$

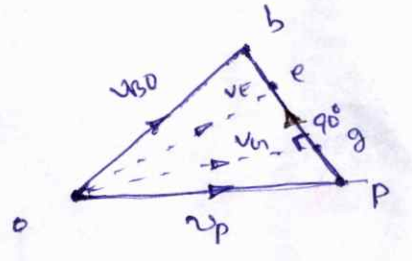
Angular velocity of CD link $\omega_{CD} = \frac{v_c}{CD} = \frac{0.385}{0.08} = 4.8\ rad/sec$

② The crank and connecting rod of the theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in clock wise direction. When it has turned 45° from the inner dead Centre position, determine 1. velocity of the piston 2. angular velocity of the connecting rod 3. Velocity of the point E on the connecting rod 1.5 m from the gudgeon pin. 4. velocity of rubbing at pins of the crank shaft, crank and cross head when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively. 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to the crank shaft.

Sol:- Given Data:- $N_B = 180$ r.p.m, $\omega_B = \frac{2\pi N_B}{60} = 18.852$ rad/sec.

$v_B = \omega_B \times OB = 18.852 \times 0.5 = 9.426$ m/sec





Velocity diagram

Sol:

$v_p = 8.15$ m/sec, $v_{PB} = 6.8$ m/sec ;

$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4$

$\frac{BE}{BP} = \frac{be}{bp} \Rightarrow be = \frac{BE}{BP} \times bp \Rightarrow v_E = \underline{\underline{oe}}$ -

(h) Velocity of Rubbing :-

$d_O = 50mm$

$d_B = 60 mm$

$d_P = 30 mm$

(i) Rubbing velocity at Crankshaft $= \frac{d_O}{2}(\omega_{BO} + 0.)$

(ii) Rubbing velocity at Crank pin $= \frac{d_B}{2}(\omega_{OB} + \omega_{BP})$

(Clock + Anticlock)

(iii) Rubbing velocity at Crosshead $= \frac{d_P}{2}(\omega_{BP} + 0)$

(5). The Position of the point 'G' on Con.Rod which has the least velocity relative to Crank shaft is determined by drawing a perpendicular from 'o' to vector bp. Since the length of 'og' will be the least, therefore the point 'g' represent the required position of 'G' on the Con.Rod.

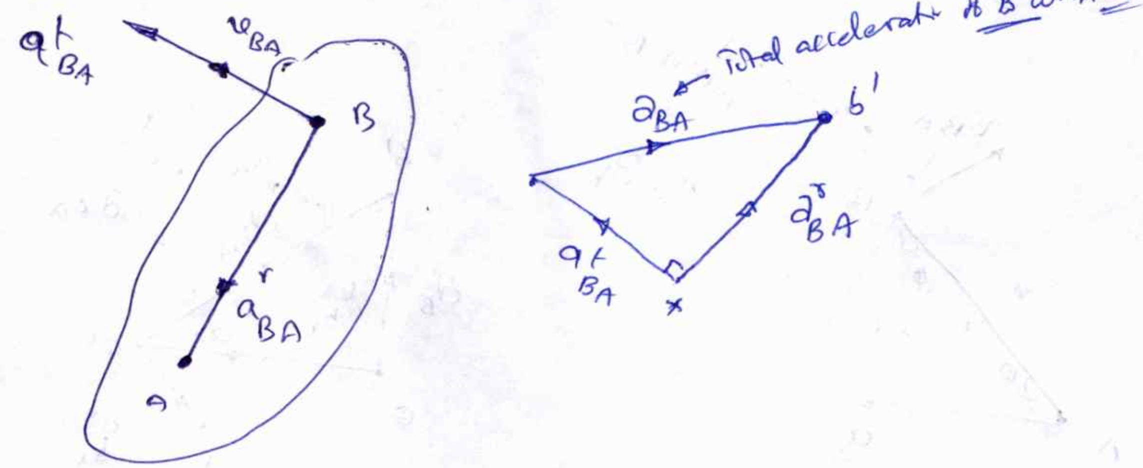
$\therefore$ $v_G = og \simeq 8\ m/s$

$\frac{bg}{bp} = \frac{BG}{BP} \Rightarrow BG = 1.47\ m$

Acceleration in Mechanisms

Acceleration diagram for a link

Consider two points A and B on a rigid link as shown in fig. Let the point B moves with respect to A, with an ~~agm~~ angular velocity of ω rad/sec and let α rad/sec² be the angular acceleration of the link AB.



We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant, has the following two components.

(1) The centripetal or radial component :- which is perpendicular to the velocity of the particle at a given instant.

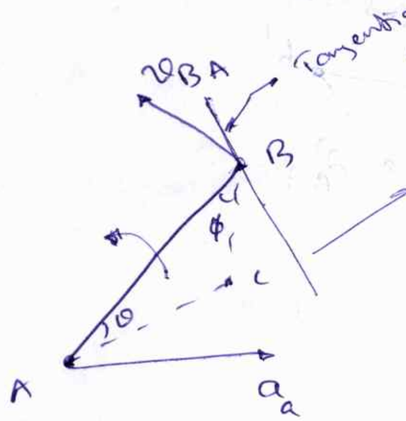
(2) The tangential component, which is parallel to the velocity of the particle at the given instant,

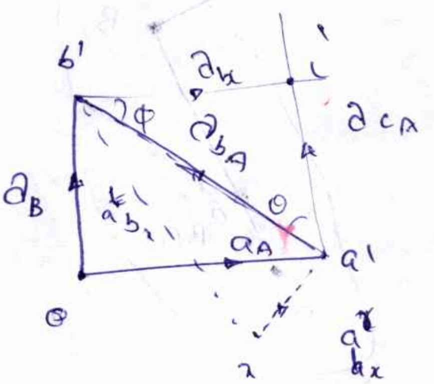
Thus for a link AB, the velocity of point B with respect to A (v_{BA}) is perpendicular to the link AB as shown in fig. Since the point B moves with respect to A with an angular velocity of ω rad/sec, therefore centripetal or radial component of the acceleration of B with respect to A.

$a^r_{BA} = \omega^2 \times$ Length of Link $AB = \omega^2 \times AB = \frac{v^2_{(BA)}}{AB}$

$a^t_{BA} = \alpha \times AB$

Acceleration of a point on a Link





$a^r_{bx} = \frac{(v_{AB})^2}{AB}$

$\boxed{a^t_{bx} = \frac{\alpha AB}{AB}}$

a_a = Absolute acceleration of point A in both magnitude and direction

a = Absolute acceleration of point B in ~~both~~ only direction

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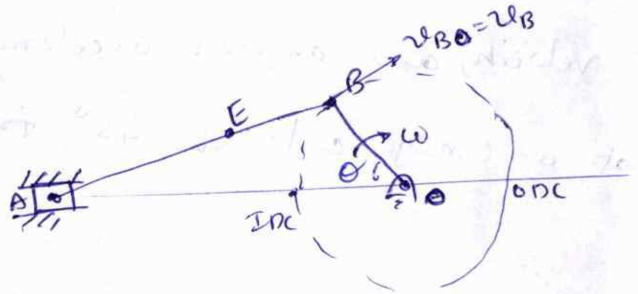
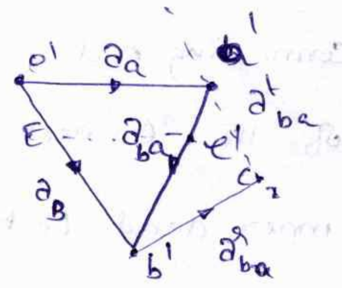
Note: the angular Acceleration of link AB is calculated by a^t_{bx}.

Acceleration in a Slider Crank Mechanism





Acceleration Diagram

(1) $a^r_{BO} = \frac{(v_{BO})^2}{OB} = a_B$

(2) $a^r_{ba} = \frac{(v_{BA})^2}{BA} = a^r_{ba}$

$Ee' = a_e$

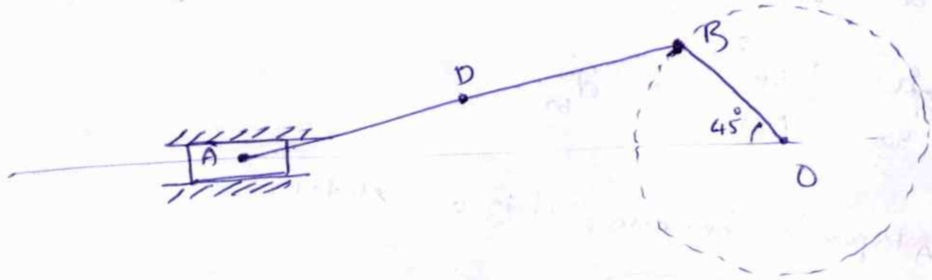
(3) a_A is parallel because it is a slider.

(4) $\boxed{\alpha_{AB} = \frac{a^t_{ba}}{AB}}$

(P) The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine

1. Linear velocity and acceleration of the midpoint of the connecting rod 2. Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Sol:-



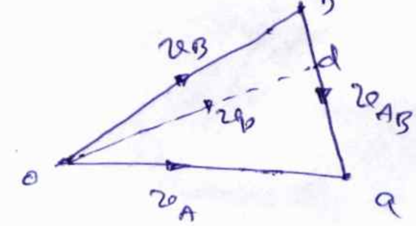
OB = 150 mm ; AB = 600 mm.

N_{OB} = 300 r.p.m ; $\omega_{BO} = \frac{2\pi N_{OB}}{60}$ = 31.42 rad/sec.

$v_{BO} = \omega_{BO} \times OB = 4.713$ m/s

By drawing velocity diagram

$v_{BA} = 3.4$ m/sec
$v_A = 4$ m/sec.
$v_D = 4.1$ m/s

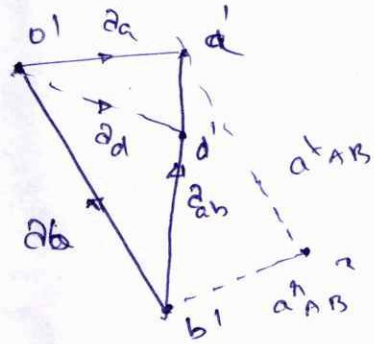


velocity diagram

Acceleration;

$$a^r_{BO} = a_b = \frac{(v_{BO})^2}{OB} = 148.1 \text{ m/sec}^2$$

$$a^r_{AB} = \frac{(v_{AB})^2}{AB} = 19.3 \text{ m/sec}^2$$



Acceleration Diagram

$a_d = 117 \text{ m/sec}^2$

$\omega_{AB} = 5.67 \quad \left(\frac{v_{AB}}{BA} \right)$

$a^t_{AB} = 103 \text{ m/sec}^2 \; ; \quad \alpha_{AB} = \frac{a^t_{AB}}{BA} = 171.67 \text{ rad/sec}^2$

Selection of a Belt Drive

The following are the various important factors upon which the selection of a belt drive depends.

1. Speed of the driving and driven shaft.
2. Speed reduction ratio.
3. Power to be transmitted.
4. Centre distance between the shafts
5. Positive drive requirements
6. Shafts layout
7. Space available
8. Service conditions

Materials

1. Leather 2. Cotton or fabric 3. Rubber belt 4. Balata

Power Transmission (Flat Belt Drives)

① Amount of Power transmitted depends up on the following factors

* The velocity of the belt.
* The Tension under which the belt is placed on the pulley
* The arc of contact between the belt and the smaller pulley.
* The conditions under which the belt is used.

Types of Belt Drives

(1) Light Drives (upto 10m/sec) (Agriculture M/C & small machine tools)

(2) Meadium Drives (10 to 22 m/sec) (Machine Tools)

(3) Heavy Drives (> 22m/sec) (Compressors & generators)

Types of Belts

(1) Flat Belts (upto 8 m) (Factories & workshops with moderate power)

(2) Ve Belts (small distances) (Same ")

(3) Circular belt or Rope. (more than 8 m) (Factories & work shops for great amount of power.

Types of Flat Belt Drives

(1) Open Belt Drive

(2) Crossed Belt drive (or) Twist belt Drive (to avoid Rubbing the distance between shafts is 20b)

(3) Quarter turn belt Drive with guid pulley (or Right angle belt drive) (Pulley width ≥ 1.4b)

(4) Belt drive with idler pulley or pulleys

(5) Compound Belt Drive.

(6) Stepped or Cone pulley drive

7. Fast and loose pulley drive.

Velocity Ratio of Belt Drive

It is the ratio between the velocities of the driver and the follower (or) driven pulley.

proof:- Let d_1 = diameter of driver

d_2 = diameter of driven.

N_1 = Speed of driver

N_2 = Speed of driven.

Length of belt passes over the driver, in one minute = $\pi d_1 N_1$

Length of belt passes over the follower, in one minute = $\pi d_2 N_2$

$\therefore$ the Length of belt passes over the driver in one minute = Length of belt passes over the follower in one minute

$$\Rightarrow \quad \pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{d_2}{d_1} \quad (or) \quad \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

If the thickness of belt is considered

then $$\boxed{\frac{N_1}{N_2} = \frac{d_2+t}{d_1+t}} \quad (or) \quad \boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

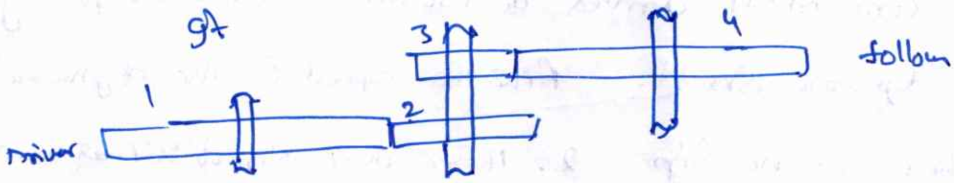
(or) $$v_1 = \frac{\pi d_1 N_1}{60}, \quad v_2 = \frac{\pi d_2 N_2}{60}$$

$$\Rightarrow \quad v_1 = v_2 \Rightarrow \boxed{d_1 N_1 = d_2 N_2}$$

$$\Rightarrow \quad \boxed{\frac{d_1}{d_2} = \frac{N_2}{N_1}}$$

Velocity Ratio of a Compound Belt Drive

Velocity Ratio $\Rightarrow$ $\dfrac{\text{Speed of last driven pulley}}{\text{Speed of first driver.}} = \dfrac{\text{Product of diameters of drivers}}{\text{Product of diameter of followers}}$



$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow \quad \frac{N_4}{N_3} = \frac{d_3}{d_4}$$

But $\quad N_3 = N_2 \Rightarrow$

$$\Rightarrow \quad N_4 = N_3 \times \frac{d_3}{d_4} = N_4 = N_1 \times \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\Rightarrow \quad \boxed{\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}}$$

Slip of the Belt

πd_1

$\cancel{\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \frac{\pi d_1 N_1}{} = v}$

Velocity of belt passing over the driver per second

slip of driver

velocity of belt $\quad v = \dfrac{\pi d_1 N_1}{60} - \dfrac{\pi d_1 N_1}{60} \times \dfrac{s_1}{100} = \dfrac{\pi d_1 N_1}{60}\left(1 - \dfrac{s_1}{100}\right)$ — ①

· velocity of the belt passing over the follower per second.

↳ slip of follower

$$\frac{\pi d_2 N_2}{60} = v - v \times \frac{s_2}{100} = v\left(1 - \frac{s_2}{100}\right) \quad — ②$$

$\therefore$ Sub ① in ②

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_2}{100} - \frac{s_1}{100} + \frac{s_1 s_2}{100}\right)$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1 + s_2}{100}\right) \quad \Bigg| \quad \Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

s = total % of s = $\%s_1 + \%s_2$

(1) An engine running at 150 r.p.m, drives a lineshaft by means of belt. The engine pulley is 750 mm diameter and the pulley on the lineshaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. There is no slip 2. There is a slip of 2% at each drive

(1500 rpm)

($S_1 = 2$; $S_2 = 2$)

(1440 rpm

Creep of Belt

When the belt passes from the slack side to tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side.

Due to ~~the~~ these changes of lenth, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering the creep the velocity ratio is given by

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(\frac{E + \sqrt{G_2}}{E + \sqrt{G_1}}\right)}$$

Where: $G_1 = G_2$ = Stress in the belt on the tight and slack side respectively

E = Young's modules of the belt material.

(P) The power is transmitted from a pulley 1000 mm diameter running at 200 r.p.m to a pulley 2250 mm diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight and slack side of the belt is 1.4 MPa and 0.5 MPa respectively. The young's modulus of the material of belt is 100 MPa.

$$\left(\left(\overset{①}{N_2} = N_1 \frac{d_1}{d_2} \right) ; \quad \overset{②}{N_2} = N_1 \frac{d_1}{d_2} \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}} \right)$$

(neglect creep) (consider creep)

$$\overset{①}{N_2} - \overset{②}{N_2} = 0.2 \text{ rpm}$$

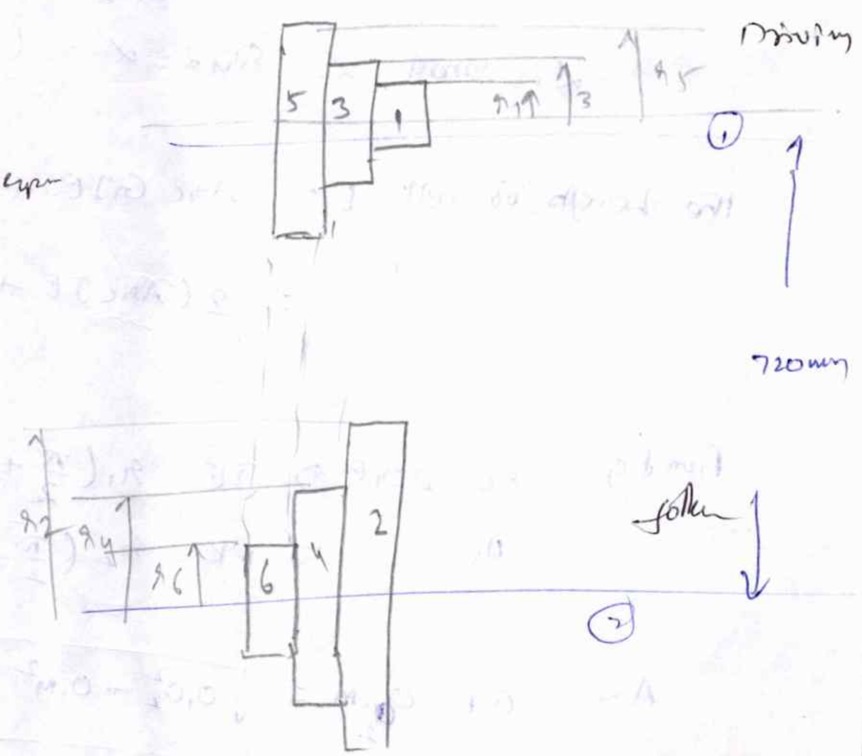
(P) A shaft which rotates at a constant speed of 160 r.p.m is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80, and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the stepped pulleys for 1. a crossed belt and 2. an open belt. Neglect belt thickness and slip.

$N_1 = N_3 = N_5 = 160$ r.p.m

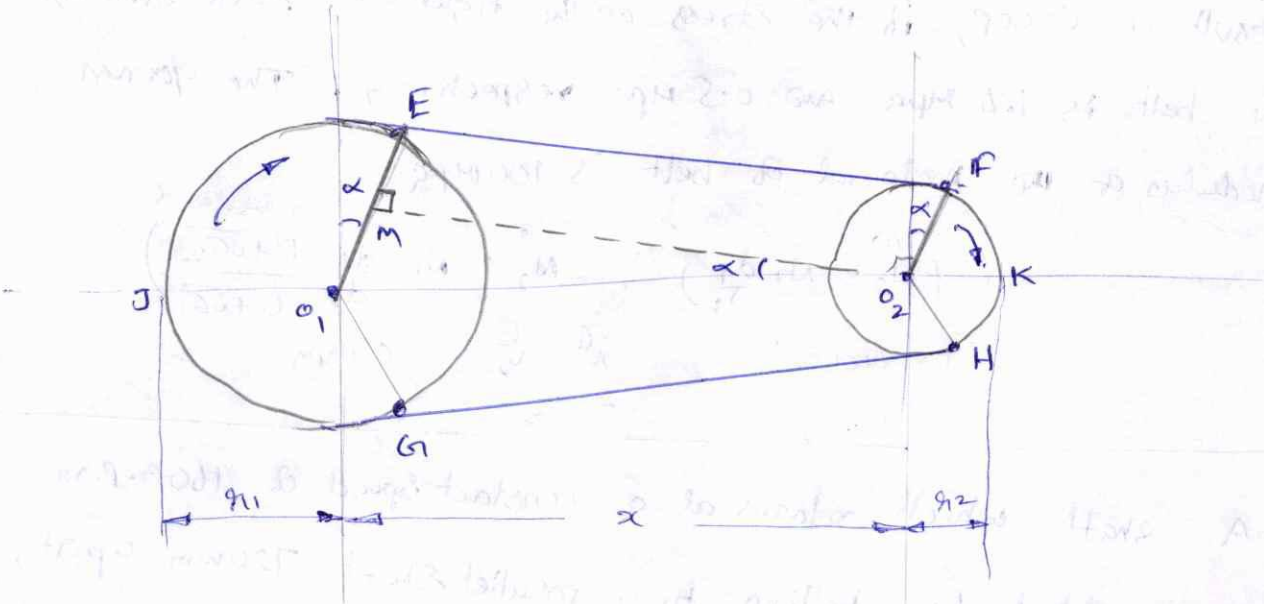
$x = 720$ mm

$N_2 = 60$; $N_4 = 80$; $N_6 = 100$ rpm

$r_1 = 40$ mm



Length of Open Belt Drive



Let $r_1 = r_2 =$ Radii of driver and follower pulleys respectively.

$x =$ Distance between the centres of two pulleys $(O_1 O_2)$

$L =$ Length of the belt.

from $\triangle(O_1 M O_2)$: $\sin\alpha = \dfrac{O_1E - ME}{O_1O_2} = \dfrac{(r_1 - r_2)}{x}$

α is small $\therefore \sin\alpha = \alpha = \dfrac{(r_1 - r_2)}{x}$

The Length of belt $L =$ Arc GJE + EF + Arc FKH + GH

$= 2(\text{Arc } JE + EF + \text{Arc } FK)$ $\left[\begin{array}{l} \because EF = GH \\ JG = JE \ \& \\ FK = KH \end{array}\right]$

from fig Arc $JO_1E \Rightarrow JE = r_1\left(\frac{\pi}{2} + \alpha\right)$

$\text{||}^{ly} \Rightarrow FK = r_2\left(\frac{\pi}{2} - \alpha\right)$

And $EF = O_2M = \sqrt{O_1O_2^2 - O_1M^2}$

$\Rightarrow O_2M = \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$

By Using Bi-nomial theorem ~~of~~ form $= x\sqrt{1-\left(\frac{1}{x^2}\right)}$

$$= x\left[1-\frac{1}{2}\left(\frac{1}{x^2}\right)+\cdots\right]$$

$$\cancel{= x}\{$$

$$\therefore x\sqrt{1-\left(\frac{r_1-r_2}{x}\right)^2} = x\left[1-\frac{1}{2}\frac{(r_1-r_2)^2}{x^2}\right]$$

$$= \boxed{x-\frac{(r_1-r_2)^2}{2x}} = O_2M = EF$$

$\therefore$ The length of belt $L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1-r_2)^2}{2x} + r_2\left(\frac{\pi}{2}-\alpha\right)\right]$

$$\Rightarrow L = 2\left[\frac{\pi r_1}{2}+\alpha r_1 + x - \frac{(r_1-r_2)^2}{2x} + \frac{\pi r_2}{2} - \alpha r_2\right]$$

$$\Rightarrow L = 2\left[\frac{\pi(r_1+r_2)}{2} + x - \frac{(r_1-r_2)^2}{2x} + \alpha(r_1-r_2)\right]$$

$$\Rightarrow L = \pi(r_1+r_2)+2x - \frac{(r_1-r_2)^2}{x} + \frac{(r_1-r_2)}{x}$$

$$\Rightarrow L = 2\left[\frac{\pi(r_1+r_2)}{2}+\alpha(r_1-r_2)+x-\frac{(r_1-r_2)^2}{2x}\right]$$

$$\Rightarrow L = \left[\pi(r_1+r_2)+2\alpha(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow L = \left\{\pi(r_1+r_2)+2\frac{(r_1-r_2)}{x}(r_1-r_2)+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow L = \left[\pi(r_1+r_2)+\frac{2(r_1-r_2)^2}{x}+2x-\frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow \boxed{L = \pi(r_1+r_2)+2x+\frac{(r_1-r_2)^2}{x}}$$

$\Rightarrow$

$$\boxed{L = \frac{\pi}{2}(d_1+d_2)+2x+\frac{(d_1-d_2)^2}{4x}}$$

Length of cross-belt Drive

$$L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

(or)

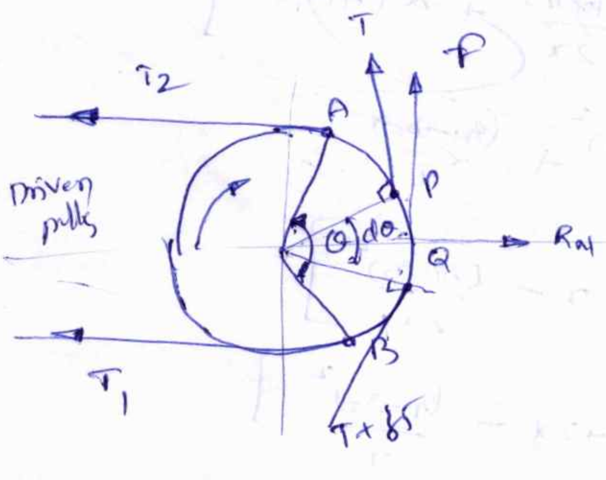
$$\boxed{L = \frac{\pi(d_1+d_2)}{2} + 2x + \frac{(d_1+d_2)^2}{4x}}$$

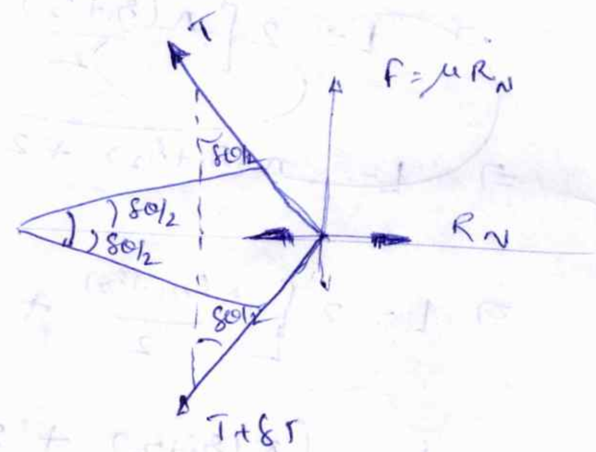
Power Transmitted by Belt Drive

Power $P = (T_1 - T_2) \times v$ (watt)

Length of –

Ratio of Driving Tensions for flat belt Drive





Let T_1 = Tension in the belt on the tight side

T_2 = Tension in the belt on the Slack side

θ = Angle of Contact in (radian) (the angle subtend by the arc AB, along which the belt touches the pulley at the Center)

Now consider a small portion of the belt PQ, subtended an angle $\delta\theta$ at the Centre of the pulley. The belt PQ is in equilibrium under the following forces.

1. Tension 'T' in the belt at P.

2. Tension $(T+\delta T)$ in the belt at Q.

3. Normal reaction R_N and

4. frictional force $f = \mu R_N$

μ = co-efficient of friction between belt & pulley.

Resolving the forces horizontally.

$$T \sin(\delta\theta/2) + (T+\delta T)\sin(\delta\theta/2) = R_N$$

sin $\sin(\delta\theta/2) \approx \delta\theta/2$ [$\because$ $\delta\theta/2$ is small]

$$\Rightarrow \quad T\times\frac{\delta\theta}{2} + T\times\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} = R_N$$

$$\Rightarrow \quad R_N = 2T\frac{\delta\theta}{2} \quad [\because \frac{\delta T \cdot \delta\theta}{2} \text{ is small}]$$

$$\Rightarrow \quad \boxed{R_N = T\delta\theta} \quad -(1)$$

Resolving the forces vertically

$$T\cos\left(\frac{\delta\theta}{2}\right) + \mu R_N = T+\delta T\cos\frac{\delta\theta}{2}$$

for small angles $\cos\left(\frac{\delta\theta}{2}\right) = 1$

$$\therefore \quad T + \mu R_N = T + \delta T$$

$$\Rightarrow \quad \boxed{R_N = \frac{\delta T}{\mu}} \quad -(2)$$

(1) = (2) $\Rightarrow$

$$T\delta\theta = \frac{\delta T}{\mu}$$

$$\mu\delta\theta = \frac{\delta T}{T}$$

Integrating the both sides w.r.t.

$$\Rightarrow \int_0^{\theta} \mu\, d\theta = \int_{T_2}^{T_1} \frac{dT}{T}$$

$$\Rightarrow \mu[\theta]_0^{\theta} = [\log T]_{T_2}^{T_1}$$

$$\Rightarrow \mu[\theta - 0] = \log_e T_1 - \log_e T_2$$

$$\Rightarrow \mu\theta = \log_e (T_1/T_2)$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

Note:- If the logarithm expressed in terms of base 10

$$\boxed{2.3 \log (T_1/T_2) = \mu\theta}$$

Determination of Angle of contact

When the two pulleys of different diameters are connected by means of an open belt as shown in fig. then the angle of contact & lap (θ) at smaller pulley to must be taken into consideration

Let r_1 = Radius of ~~smaller~~ larger pulley

r_2 = Radius of smaller pulley

x = Distance between the centers of two pulley.

For open Belt Drive $\theta = (180 - 2\alpha) \frac{\pi}{180°}$

where $\alpha = \sin^{-1}\left(\frac{r_1 - r_2}{x}\right)$

For cross Belt Drive $\theta = (180 + 2\alpha) \frac{\pi}{180°}$

where $\alpha = \sin^{-1}\left(\frac{r_1 + r_2}{x}\right)$

(P) A casting weighing 9KN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20r.p.m. The other end of the rope is pulled by a man. The co-efficient of friction is 0.25. Determine 1. The force required by the man (T_2) (2) Power to raise the casting

$$\left(\theta = 2.5 \times 2\pi = 5\pi \text{ rad}; \quad T_2 = 176.47 \text{ N}. \quad P = 2.772 \text{ KW} \right)$$

(P) Two pulleys, one 450mm diameter and other 200mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.

What power can be transmitted by the belt when the larger pulley rotates at 200rev/min, if the maximum permissible tension in the belt is 1KN, and the co-efficient of friction between the belt and pulley is 0.25?

(P) A shaft rotating at 200rpm, drives another shaft at 300r.p.m and transmits 6KW through a belt. The belt is 100mm wide and 10mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5m in diameter. Calculate the stress in the belt, if it is 1. An open belt drive 2. a cross-belt drive. Take $\mu = 0.3$

(P) Find the power transmitted by a belt running over a pulley of 600mm diameter at 200r.p.m. The co-efficient of friction between the belt and the pulley is 0.25, angle of lap 160° and the maximum Tension in the belt is 2500 N.

(Ans P = 7690W = 7.69KW

(P) A leather belt is required to transmit 7.5 KW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the co-efficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather is 1 Mg/m3 and Thickness of belt is 10mm, Determine the width of the belt taking centrifugal tension into account.

$\boxed{b = 65.8\text{mm}}$

($T_{max} = T_1 + T_C$; $T_m = \sigma \times b \times t$)

$T_C = mv^2$ $\boxed{m = \rho \times A \times l}$

$\boxed{m = 10b}$

$T_C = 2468b$ $T_1 = 824.6$

$T_{max} = 15000b$

(P) Determine the width of a 9.75mm thick leather belt required to transmit 15KW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300mm. The driven pulley runs at 300 r.p.m and the distance between the centre of two pulleys is 3 meters. The density of leather is 1000 Kg/m3. The maximum allowable stress in the leather is 2.5 MPa. The co-efficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt. (b = 80mm)

($\frac{N_2}{N_1} = \frac{d_1}{d_2}$)

$d_2 = 0.9m$

$\theta = 180 - 2\alpha$

$\alpha = \sin^{-1}\left(\frac{r_2 - r_1}{x}\right)$

Max Tension in belt

$T = $ max Stress $\times$ Area

$T = \sigma \times b \times t$

$T_G = T = T_1 + T_C$

(b) An open belt drive connects two pulleys 1.2 m and 0.5 m diameter, on parallel shafts '4' meters apart. The mass of the belt is 0.9 kg/m length and the maximum tension is not to exceed 2000 N. The co-efficient of friction is 0.3. The 1.2 m pulley, which is the driver, runs at 200 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, $P(T_1 - T_2) v$ (13.78 kW) and power lost in friction $P_I - P_o$ (0.87 kW). What is the efficiency (η) of the drive.

$$\left[P_I = \frac{2\pi N_1 T_D}{60} \ (13.78\,kW) \ ; \quad P_o = \frac{2\pi N_2 T_{follo}}{60} \ (12.91\,kW) \right]$$

$$\eta = \frac{P_I - P_o}{P_I} * 100 = 93.7\%$$

$T_D = (T_1 - T_2)\, r_d$ (657.6 N-m)

$T_f = (T_1 - T_2)\, r_f$ (274 N-m)

Insta Team

(p) In a flat belt drive, the initial tension is 2000 N. The co-efficient of friction between the belt and the pulley is 0.3 and the angle of lap on the smaller pully is 150°. The smaller pulley has a radius of 200 mm and rotates at 500 r.p.m. Find the power in kW transmitted by the belt.

An (15.7 kW)

$T_0 = \frac{T_1 + T_2}{2}$

$\frac{T_1}{T_2} = e^{\mu\theta}$

$T_1 - T_2 \times v$

Max tension $T_m = T_{t_1} + T_C$

Centrifugal Tension

m = mass per unit length (kg)

(T_c acts on both sides)

$v =$

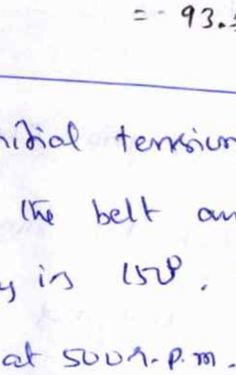
$r =$

$T_C =$ on both.

$m_b = m r d\theta$

$F_C = \frac{m_b v^2}{r} = m r d\theta \cdot \frac{v^2}{r}$

$F_C = m\, d\theta\, v^2$



length of belt at PQ $= r\, d\theta$

mass of belt $m = m r d\theta$.

$F_C = \frac{m v^2}{r}$

$F_C = T_C \sin\frac{d\theta}{2} + T_C \sin\frac{d\theta}{2}$

$m\, d\theta\, v^2 = 2 T_C \frac{d\theta}{2} \Rightarrow \boxed{T_C = m v^2}$

$\boxed{T_{max} = T_1 + T_C}$

Condition for Max Power Transmission

$P = (T_1 - T_2) \cdot v = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) \cdot v$

$\frac{dP}{dv} = 0$;

$P = T_1 C \cdot v = (T_{max} - T_C) \cdot C \cdot v$

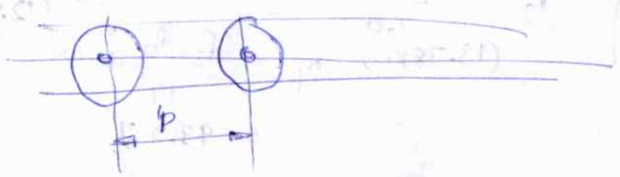
$P = (T_m - m v^2) \cdot C \cdot v$;

$\boxed{T_{max} = \frac{T_C}{3}}$

CHAIN DRIVES

Terms Used in the Chain Drive

(1) Pitch of the Chain:- It is the distance between the hinge Centre of a link and the corresponding hinge centre of the adjacent link. It is usually denoted by 'p'.



(2) Pitch circle diameter of the Chain Sprocket:- It is the diameter of the circle on which the hinge centres of the Chain lie, when the chain is wrapped round a sprocket as shown in fig.

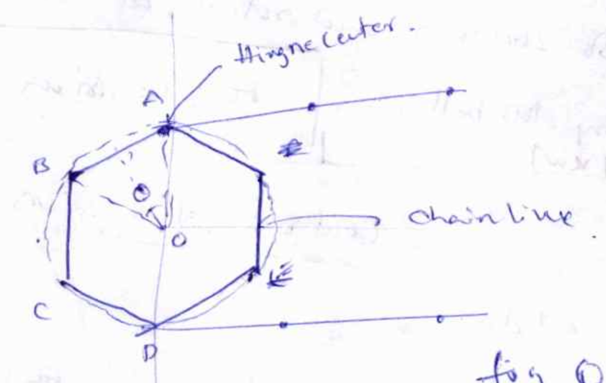


fig ①

~~From~~ the points A, B, C, D are the hinge centers of ~~Since~~ the chain and the circle drawn through these centers is called pitch circle and its diameter is known as pitch circle diameter.

Relation between the Pitch and pitch circle diameter

From the fig(1) the pitch length is a chord AB. Consider the pitch length 'AB' of the chain subtending an angle 'θ' at the center of the sprocket.

Let d = Diameter of the pitch circle

T = Number of teeth on the sprocket

From fig:

$$\sin\theta/2 = \left(\frac{AB}{2}\right)/OA$$

$$\Rightarrow AB = 2\,OA \sin(\theta/2)$$

$$[\because OA = r;\ 2OA = d]$$

$$\therefore \text{Pitch } \boxed{(p) = d \times \sin\theta/2}$$

(or)

$$\text{Pitch circle diameter } \boxed{d = p \times \operatorname{cosec}(\theta/2)}$$

If T = No. of Teeth on the sprocket

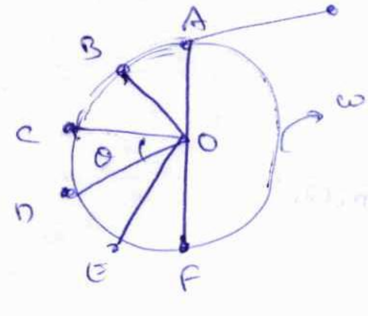
$$\therefore \text{angle } \boxed{\theta = \frac{360°}{T}}$$

$$\therefore p = d \sin\left(\frac{360}{2T}\right) \Rightarrow \boxed{p = d \sin\left(\frac{180}{T}\right)} *$$

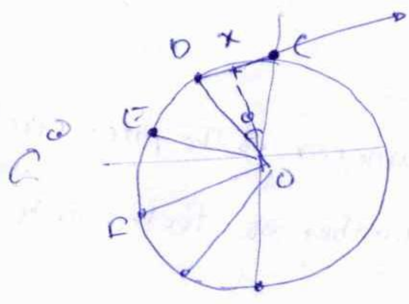
Or

$$\boxed{d = p \times \operatorname{cosec}\left(\frac{180}{T}\right)}$$

Relation between the Chain Speed and Angular Velocity of Sprocket



(a)



(b)

$\cos \theta/2 = \frac{OX}{OC}$

For angular position of sprocket as shown in fig (a)

$$\boxed{v = \omega_0 OA}$$

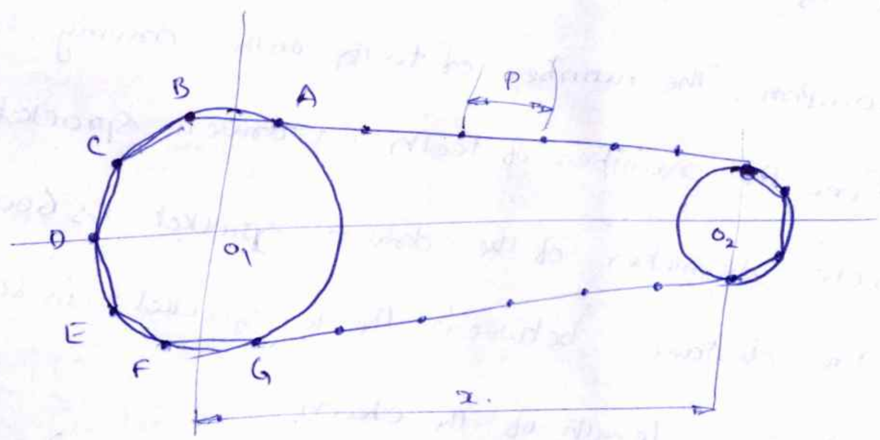
For angular position of sprocket as shown in fig (b)

$$v = \omega \times OX \Rightarrow v = \omega \times OC \times \cos \theta/2$$

$[\because OA = OC$

$$\Rightarrow \boxed{v = \omega \times OA \cos(\theta/2)}$$

Length of chain



from :- Length of open belt drive formul

$$L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}$$

from

$$p_1 = d_1 \sin\left(\frac{180}{T_1}\right) \quad \Big| \quad p_2 = p_1 = d_2 \sin\left(\frac{180}{T_2}\right)$$

$$r_1 = \frac{p}{2}\operatorname{Cosec}\left(\frac{180}{T_1}\right) \quad \Big| \quad r_2 = \frac{p}{2}\operatorname{Cosec}\left(\frac{180}{T_2}\right)$$

$$\therefore \quad L = \pi\left[\frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_1}\right) + \frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_2}\right)\right] + 2x + \frac{\left[\left(\frac{p}{2}\operatorname{cosec}\left(\frac{180}{T_1}\right)\right) - \left(\frac{p}{2}\operatorname{cosec}\frac{180}{T_2}\right)\right]^2}{x}$$

$$\Rightarrow L = \left[\frac{p}{2}(T_1+T_2) + 2x + \frac{p^2\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{2}\right)\right)^2}{4x}\right]$$

$$\Rightarrow L = p\left[\frac{(T_1+T_2)}{2} + \frac{2x}{p} + \frac{p\left(\operatorname{cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{T_2}\right)\right)^2}{4x}\right]$$

$$\Rightarrow L = \boxed{\text{If} \quad x = mp \quad \& \quad \frac{x}{p} = m}$$

then

$$L = p\left[\frac{(T_1+T_2)}{2} + 2m + \frac{\left(\operatorname{Cosec}\left(\frac{180}{T_1}\right) - \operatorname{cosec}\left(\frac{180}{T_2}\right)\right)^2}{4m}\right]$$

$$\boxed{L = p \cdot K}$$

where K = Multiplying factor $= \left[\left(\frac{T_1+T_2}{2}\right) + 2m + \frac{\left(\operatorname{Cosec}\left(\frac{180}{T_1}\right) - \operatorname{Cosec}\left(\frac{180}{T_2}\right)\right)^2}{4m}\right]$

(P) A chain drive is used for reduction of speed from 240 r.p.m to 120 r.p.m. The number of teeth on the driving sprocket is 20. Find the number of teeth on driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and center to centre distance between the two sprockets is 800 mm, determine the pitch and length of the chain.

A

(Hint :- $\frac{N_1}{N_2} = \frac{T_2}{T_1}$, $d_1 = P_1 \operatorname{cosec}\left(\frac{180}{T_1}\right)$)

$T_2 = 60$; $p = 47.1\,mm$

$\boxed{L = 3.0615\,m \; ; \quad K = 64.86 \simeq 65}$

-V- Belt

$$\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}}$$

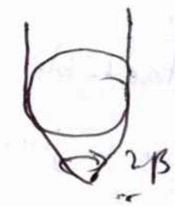
(P) A compressor, requiring 90 kW is to run at about 250 r.p.m. The drive is by -V- belts from an electric motor running at 750 r.p.m. The diameter of the pulley on the compressor shaft must not be greater than 1 meter while the centre distance between the pulleys is limited to 1.75 meter. The belt speed should not exceed 1600 m/min.

Determine the number of -V- belts required to transmit the power if each belt has a cross-sectional area of 375 mm^2, density 1000 kg/m^3 and an allowable tensile stress of 2.5 MPa. The groove angle of the pulley is 35°. The co-efficient of friction between the belt and the pulley is 0.25. Calculate also the length required of each belt.

(no of belts $n = 5.6 \simeq 6$; $L_0 = 5.664\,m$

Rope Drives

Types of Rope drives



(1) fibre ropes (upto 60 m)   (2) wire Ropes (upto 150 m)

Ratio of belt Tension $\frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\beta}}$

2β = Groove angle

β = Semigroove angle.

(P). A rope drive transmitts 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160. the angle of groove is 45°. the co-efficient of friction 0.28. The mass of rope 1.5 kg/m and the allowable tension in each rope (Tmax) 2400 N. Find the number of ropes required.

$(v = \frac{\pi d N}{60} = 18.85 \, m/s \; ; \; T_c = 533 N$

$T_1 = T_{max} - T_c$

no. of ropes = $\frac{\text{Total Power of } \cancel{\text{each rope}}}{\text{Power of each rope}} = \frac{600}{30.67} \approx 19.58 \approx 20$

Classification of chains

1. Hoisting and hauling chains (or Crane Chains)
2. Conveyor Chains (or Tractive ~~tra~~ chains)
3. Power Transmitting chains (or Driving chains)

(1) Hoisting and hauling chains

Chain with oval link

Chain with Square link

Conveyor chains

(1) Detachable & hook joint type chain

(2) Closed joint type chain

Power Transmitting Chains

1. Block chain
2. Bush Roller chain
3. Inverted tooth & Siled chain

Additional Problems

(P) A pulley is driven by a flat belt, the angle of lap being 120°. The belt is 100 mm wide by 6mm thick and density $1000\ kg/m^3$. If the co-efficient of friction is 0.3 and the maximum stress in the belt is not to exceed 2MPa, find the greatest power which the belt can transmit and the corresponding speed of the belt.

[Ans: $T_{max} = 1200N$; $m = 0.6 kg/m$; $v = 25.82\ m/sec$; $T_c = \frac{T_{max}}{3} = 400N$
$T_1 = T_{max} - T_c = 800N$; $T_2 = 425.5N$; $P = (T_1 - T_2) \times v = 9.67\ kW$]

Coriolli's Component of Acceleration

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of acceleration must be calculated.

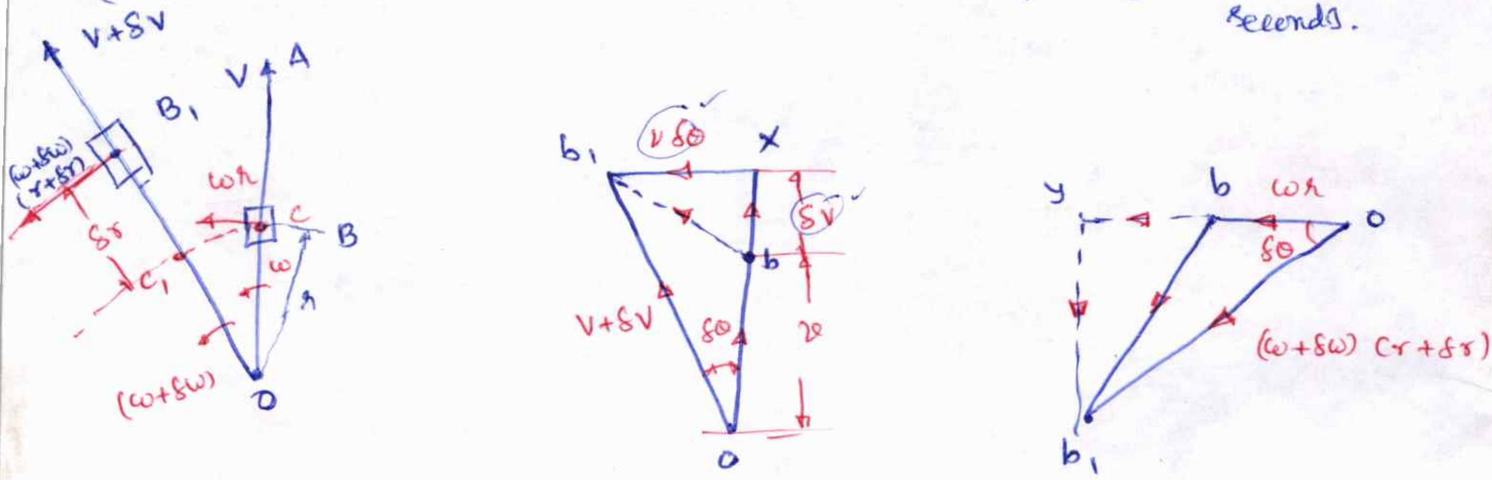
Consider a link 'OA' and a slider 'B' as shown in fig. The slider 'B' moves along the link 'OA'. The point 'C' is the coincident point on the link 'OA'.

Let ω = Angular velocity of the link 'OA' at time 't' seconds.

v = Velocity of the slider 'B' along the link 'OA' at time 't' seconds

$\omega . r$ = Velocity of the slider 'B' with respect to 'O' at time 't' seconds

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)(r + \delta r)$ = Corresponding values at time $(t + \delta t)$ seconds.



(a) (b) (c)

Fig: Coriolis Component of Acceleration.

Let us now find out the acceleration of the slider 'B' with respect to 'O' and with respect to its coincide point 'C' lying on the link OA.

from Fig (b):- the vector 'bb_1' represents the change in velocity in time δt sec.
the vector 'bx' represents the component of change of velocity bb_1 along direction 'OA' (Radial directn) and the vector 'xb_1' its component in the direction perpendicular to 'OA' (i.e Tangential direction).

$\therefore \quad bx = ox - ob = (v + \delta v) \times \cos\delta\theta - (v)$ (Acting radially outward)

$\text{lly} \quad = (v + \delta v) - (v) = \delta v$ ($\because$ $\cos\delta\theta = 1$ small angle)

$\text{lly} \therefore \quad xb_1 = (v + \delta v)\sin\delta\theta = (v + \delta v)\delta\theta$ [$\because$ small angle $\sin\theta = \theta$]

$xb_1 = v\delta\theta + \delta v\delta\theta \simeq \overleftarrow{v\delta\theta}$ [$\because$ product of small × small = very small its value] from fig (d)

$$yb_1 = (\omega+\delta\omega)(r+\delta r)\sin\delta\theta \downarrow$$
$$= [\omega r + \omega\delta r + r\delta\omega + \delta r\delta\omega] \times \delta\theta \quad [\text{small angle } \sin\delta\theta = \delta\theta]$$
$$= \omega r\delta\theta + \omega\delta r\delta\theta + r\delta\omega\delta\theta + \delta r\delta\omega\delta\theta \downarrow$$
$$= \omega r\delta\theta \downarrow \quad [\because \text{ product of small quantities is neglected}]$$
[Acting radially inward]

$$by = oy - ob = (\omega+\delta\omega)(r+\delta r)\cos\delta\theta - \omega r \quad [\text{small angle } \cos\delta\theta = 1]$$
$$= \cancel{\omega r} + \omega\delta r + r\delta\omega + \delta\omega\delta r - \cancel{\omega r}$$
$$= r\delta\omega + \omega\delta r$$

$\therefore$ Total component of change of velocity along Radial direction

$$= bx - yb_1 = (\delta v - \omega r\delta\theta) \uparrow$$

$\therefore$ Radial component of the Acceleration of Slider 'B' with respect to 'O' on the link 'OA' acting Radially outward from 'O' to 'A'.

$$a^r_{BO} = \lim_{\delta t\to 0} \frac{(\delta v - \omega r\delta\theta)}{\delta t} = \frac{dv}{dt} - \omega r\frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2 r \uparrow$$

$\therefore$ Also, the Total component of change of velocity along Tangential direction

$$= xb_1 + by = v\delta\theta + r\delta\omega + \omega\delta r$$

$\therefore$ Tangential component of the acceleration of the slider 'B' with respect 'O' on the link 'OA' acting perpendicular to 'OA' and towards left.

$$a^t_{BO} = \lim_{\delta t\to 0} \frac{(v\delta\theta + r\delta\omega + \omega\delta r)}{\delta t} = v\frac{d\theta}{dt} + r\frac{d\omega}{dt} + \omega\frac{dr}{dt} = v\omega + r\alpha + \omega v$$

$$\Rightarrow \boxed{a^t_{BO} = 2v\omega + r\alpha}$$

Now:- the Radial component of acceleration of the coincident point 'C' with respect to 'O' acting in the direction from 'C' to 'O'

$$a^r_{CO} = \omega^2 r \downarrow$$

the Tangential Component of the Slider 'B' with respect to 'O', acting in the direction perpendicular to 'CO' and towards left

$$a^t_{CO} = \alpha r \uparrow$$

$\therefore$ The Radial component of the Slider 'B' with respect to the coincident point 'C' on the link 'OA' acting radially outwards

$$a^r_{BC} = a^r_{BO} + a^r_{CO} = \left(\frac{dv}{dt} - \omega^2 r\right) + (\omega^2 r) \quad [(-)\text{ve Resultant is opposite}]$$

$$\boxed{a^r_{BC} = \frac{dv}{dt} \uparrow}$$

$\therefore$ The Tangential Component of the Slider 'B' with respect to the coincident point 'C' on the link 'OA' acting in the direction perpendicular to 'OA' and towards left.

$$a^t_{BC} = a^t_{BO} - a^t_{CO} = (2\omega v + \alpha r) - \alpha r = \overleftarrow{2\omega v}$$

∴ This Tangential component of acceleration of the slider 'B' with respect to the coincident point 'C' on the link is known as "Coriollis component of Acceleration" and is always perpendicular to the link.

∴ Coriollis component of the acceleration of 'B' with respect to 'C'

$$\boxed{a^c_{BC} = a^t_{BC} = \underline{\underline{2\omega v}}}$$

Note:- In the above discussion, the anticlockwise direction for ω and radially outward direction for 'v' are taken as positive. It may be noted that the direction of coriollis component of acceleration changes sign, if either 'ω' or 'v' is reversed in the direction. But if the coriollis component component of acceleration will not be changed if the sign of both 'ω' and 'v' are reversed in the direction.