21/01/2023 UNIT-I

Introduction to heat transfer

* Heat Transfer can be defined as the transmission of energy from one region to another region due to Temperature difference.

Modes of Heat Transfer

* Conduction
* Convection
* Radiation

⇒ Heat Conduction is a mechanism of Heat transfer from a region of high temperature to a region of low temperature within a medium. [solid, liquid & Gases] or between difference medium in direct physical Contact

⇒ In Conduction Energy exchanges takes place by the kinematic motion or direct Impact of molecules. Pure Conduction is found out in Solids.

⇒ Convection : Convection is a process of Heat transfer that will occur b/w a solid surface and a fluid medium, when they are at different temperatures.

⇒ Convection is possible only in the presence of fluid medium.

⇒ Radiation : The Heat transfer from one body to another body without any transmitting medium is known as "Radiation"

⇒ It is an "electromagnetic wave phenomena"

Basic Laws of Heat Transfer

* First Law of Thermodynamics : It states that when system undergoes a cyclic process, net Heat transfer equal net work transfer

$$\oint Q = \oint W$$

* Second Law of Thermodynamics :

Kelvin's Plank statement : It is impossible to construct an engine working on an cyclic process when converts all the heat energy supplied to it into an equivalent amount of useful work.

Clasius Statement : It stated that heat can flow from hot body to Cold body without any external aid, but heat can't be flow from Cold body to hot body without any external aid.

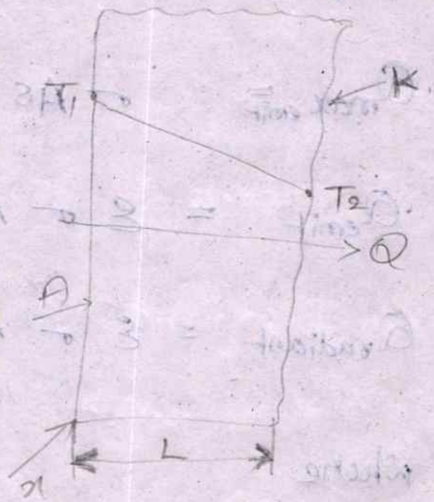
Law of Conservation of Mass

⇒ This law is used to determine the parameters of flow.

Fourier law of Equations

⇒ Rate of heat conduction is proportional to the area measured Normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q \propto -A \frac{dT}{dx}$$



$$Q = -KA \frac{dT}{dx}$$

$$Q = \frac{-KA\ (T_1 - T_2)}{L}$$

where

K = Thermal conductivity - W/mK

A = Area - m^2

$\frac{dT}{dx}$ = Temperature gradient Kel/m

The negative sign indicates that the heat flow in a direction alog which there is a decrease in temperature.

Newton's Law of cooling or Convection

⇒ Heat transfer from the moving fluid to a solid surface is given by a equation

$$Q = h A_s (T_{surface} - T_{surrounding})$$

$$= h A (T_w - T_\infty)$$

where, h = local heat transfer coefficient - W/m^2K

Radiation

⇒ Maximum rate of radiation that can be emitted by the surface.

$$Q_{max\ emit} = \sigma A_s T_s^4$$

$$Q_{emit} = \varepsilon \sigma A_s T_s^4$$

$$Q_{radiant} = \overset{Epsal}{\varepsilon} \sigma A_s (T_s^4 - T_{sur}^4)$$

where

σ = stephen Bolzmen constant 5.67×10^{-8} W/m^2K^4

T_s = surface temperature

T_{sur} = surrounding temp

ε = Epsal (Emisivity)

(1)
Sol ~~Sol~~ Given data

$A = 6 \times 8 = 48\,m^2$

$L = 0.25\,m$

$K = 5.08$

$T_1 = 4^\circ C$

$T_2 = 15^\circ C$

time = 10 hrs

Amount = \$ 0.08/kwh.

a) $Q = -KA\dfrac{dT}{dx}$

$= -KA\dfrac{T_1 - T_2}{L}$

$= \dfrac{-5.08 \times 48 \times (4-15)}{0.25} = 10728.96\,W$

$= 10.728\,KW.$

b) $Q_t = Q \times time$

$= 10.728 \times 10$

$= 107.28\,kwh$

cost = Amount $\times Q_t$

$= 0.08 \times 107.28$ kwh

$= \$\ 8.58$

(2)
Sol Given data

~~A = 2×3~~ 30 m

(2) sol Given data

1m = 100cm

1cm = 0.01 m

0.3 =

$L = 2m$

$D = 0.003m$

$T_\infty = 15^\circ C$

$T_w = 152^\circ C$

$V = 60V$

$h = ?$

$I = 1.5A$

$Q = hA(T_w - T_\infty)$

$Q = V \times I = 60 \times 1.5 = 90W$

$A = \pi DL = \pi \times 0.003 \times 2 = 0.01885$

$90 = h \times 0.01885 \times (152 - 15)$

$h = 34.9 \ w/m^2k.$

(3) sol Given data

Winter $T_{sur} = 10^\circ C + 273 = 283K$

Summer $T_{sur} = 25 + 273 = 298 K$

$Q = \varepsilon \sigma A_s (T_s^4 - T_{sur}^4)$

$A_s = 1.4 m^2$

$T_s = 30^\circ C + 273 = 303K$

Emmissitivity of a person $\varepsilon = 0.95$

$\sigma = 5.67 \times 10^{-8} \ w/m^2 k^4$

$Q_{rad, summer} = 0.95 \times 5.67 \times 10^{-8} \times 1.4 \times (303^4 - 298^4)$

$= 40.9 W$

Q_{rad}, winter $= 0.95 \times 5.67 \times 10^{-8} \times 1.4 (303^4 - 283^4)$

$= 152 W.$

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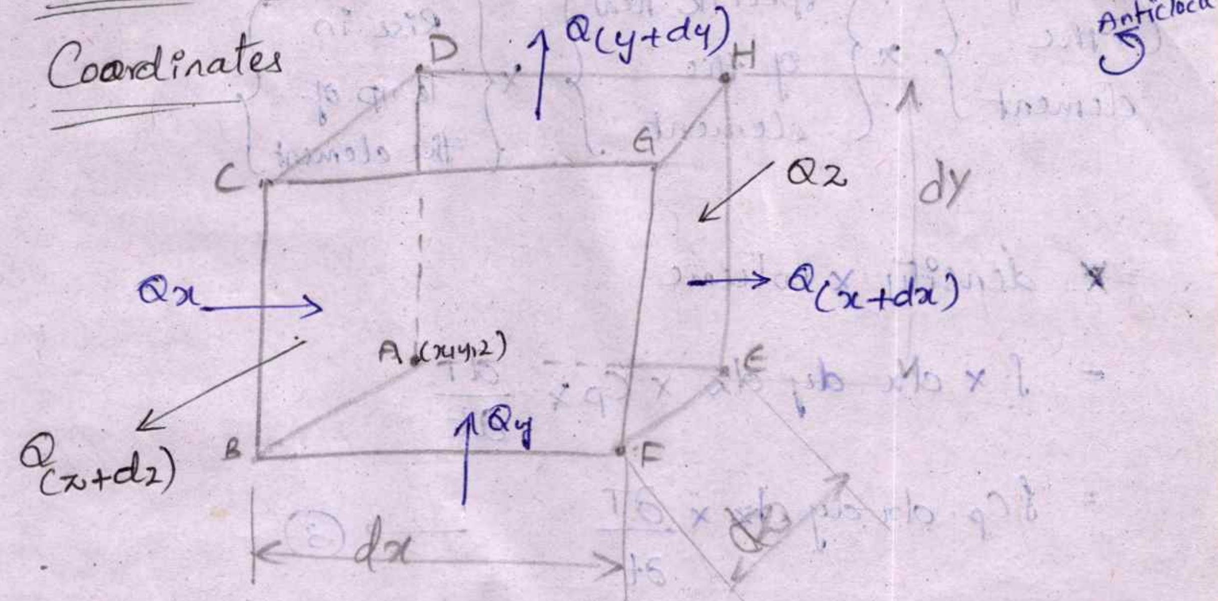
General Heat conduction Equation in Cartesian Coordinates



~~Net heat conducted into element from~~ Consider a small rectangular element of sides dx, dy, dz as shown in fig. The energy balance of this rectangular element is obtained from first law of Thermodynamics.

Net heat conducted in to element from all the coordinate directions (1)	+	Heat generated with in the elements (2)	=	Heat stored in the element. (3)

⇒ Heat generated with in the Element

$$Q_g = \dot{q}\, dx\, dy\, dz \Rightarrow ②$$

⇒ Heat stored in the Element ③ ⇒

$$\begin{Bmatrix} \text{mass of} \\ \text{the} \\ \text{element} \end{Bmatrix} \times \begin{Bmatrix} \text{specific heat} \\ \text{of the} \\ \text{element} \end{Bmatrix} \times \begin{Bmatrix} \text{Rise in} \\ \text{Temp of} \\ \text{the element} \end{Bmatrix}$$

= density x volume

$$= \rho \times dx\, dy\, dz \times C_p \times \frac{\partial T}{\partial t}$$

$$= \rho C_p \cdot dx\, dy\, dz \times \frac{\partial T}{\partial t} \longrightarrow ③$$

⇒ Net Heat conducted into the element in X-direction

Let q_x be the heat flux in the direction of face in a A, B, C, D and q_{x+dx} be the heat flux in a direction of face EFGH.

The rate of heat flow into the element in x-direction through the face ABCD

$$Q_x = q_x\, dy\, dz$$

$$= -k_x \frac{\partial T}{\partial x} dy\, dz \longrightarrow ⓐ$$

The rate of heat flow out of the element in x-direction through the face EFGH

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x)\,dx \longrightarrow (b)$$

Net heat conducted into the element in x-direction

$$Q_x - Q_{x+dx} = \cancel{Q_x} - \left[\cancel{Q_x} + \frac{d}{dx}(Q_x)\,dx\right]$$

$$= -\frac{d}{dx}\left(-k_x \frac{dT}{dx}\,dy\,dz\right)dx$$

$$= \frac{\partial^2 T}{\partial x^2} k_x\,dx\,dy\,dz$$

$$= k_x\,dx\,dy\,dz\,\frac{d^2T}{dx^2}$$

Similarly,

$$Q_y - Q_{y+dy} = k_y\,dx\,dy\,dz\,\frac{d^2T}{dy^2}$$

$$Q_z - Q_{z+dz} = k_z\,dx\,dy\,dz\,\frac{d^2T}{dz^2}$$

Considering the material is isotropic

$$k = k_x = k_y = k_z$$

Net heat conducted into the element, from all the coordinate directions

$$\left(\frac{d^2T}{dx^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) k\,.\,dx\,dy\,dz \longrightarrow (1)$$

$$\left[\left(\frac{d^2T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)k+\dot{q}\right]\cancel{dx\,dy\,dz} = \rho c_p \frac{dT}{dt}\cancel{dx\,dy\,dz}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t}$$

$$= \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\left(\because \alpha = \frac{k}{\rho c_p} = \text{thermal diffusivity}\right)$

Case (i) : No Heat source

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Case (ii) : Steady state Conditions

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k} = 0$$

$$\nabla^2 T + \frac{\dot{q}}{k} = 0$$

poisoins equation

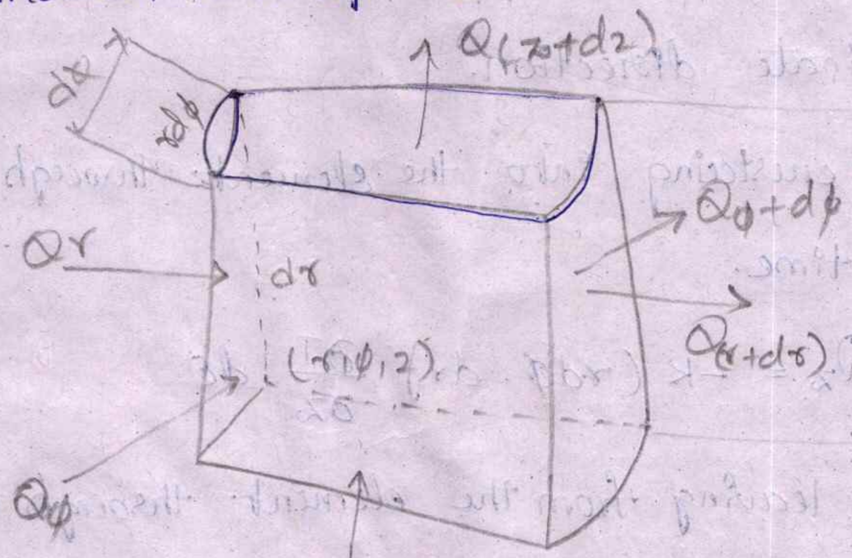
Case (iii) : One dimensional study state Heat conduction.

$$\frac{d^2T}{\partial x^2}+\frac{q}{k} = 0$$

Case (iv) unsteady one dimensional without Internal Heat generation. $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

General Heat Conduction equation in Cylindrical coordinates

The general heat conduction in Cartesian coordinate derived in the previous section is used for solids with rectangular boundaries like square, cube, slab etc. But Cartesian coordinate system is not applicable for, solid, cylinders, cones, spheres etc.



Consider a small cylindrical element of sides $dr, d\phi$, and dz as shown in fig.

⇒ The volume of the element $dv = rd\phi . dr . dz$

⇒ Let us assume that thermal conductivity (k), Specific heat (C_p) and density (ρ) are constant.

⇒ The energy balance of this cylinder is first Law of Thermodynamics.

Net heat conducted into the element ~~from~~ all the coordinate directions (+) Heat generated with in the element = Heat stored in the element.

⇒ Heat Generated with in the element

$$Q = \dot{q}\,(r\,d\phi\; dr\; dz)\; d\theta$$

⇒ Heat stored in the element

$$\rho \times r\,d\phi\; dr\; dz \times C_p \cdot \frac{\partial T}{\partial \theta} \cdot d\theta$$

⇒ Net heat conducted into element from all the coordinate direction.

* Heat enesteeing into the element through r & ϕ plane in time.

$$Q_z = -k\,(r\,d\phi \cdot dr)\,\frac{\partial T}{\partial z} \cdot d\theta$$

* Heat leaving from the element through r & ϕ plane in time $d\theta$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z)\,dz$$

⇒ Net heat conducted into the element through r & ϕ plane in time $d\theta$.

$$= Q_z - Q_{z+dz}$$

$$= \frac{\partial}{\partial z}(Q_z)\,dz$$

$$= \frac{\partial}{\partial z}\left[k\,(r\,d\phi\; dr)\left(\frac{\partial T}{\partial z}\right)d\theta\right] dz$$

$$= k\,(r\,d\phi\; dr\; dz)\,\frac{\partial^2 T}{\partial z^2} \cdot d\theta.$$

+

Similarly,

$= Q_r - Q_{r+dr}$

$= d\theta . K(dr . r d\phi . dz)\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right)$

Similarly,

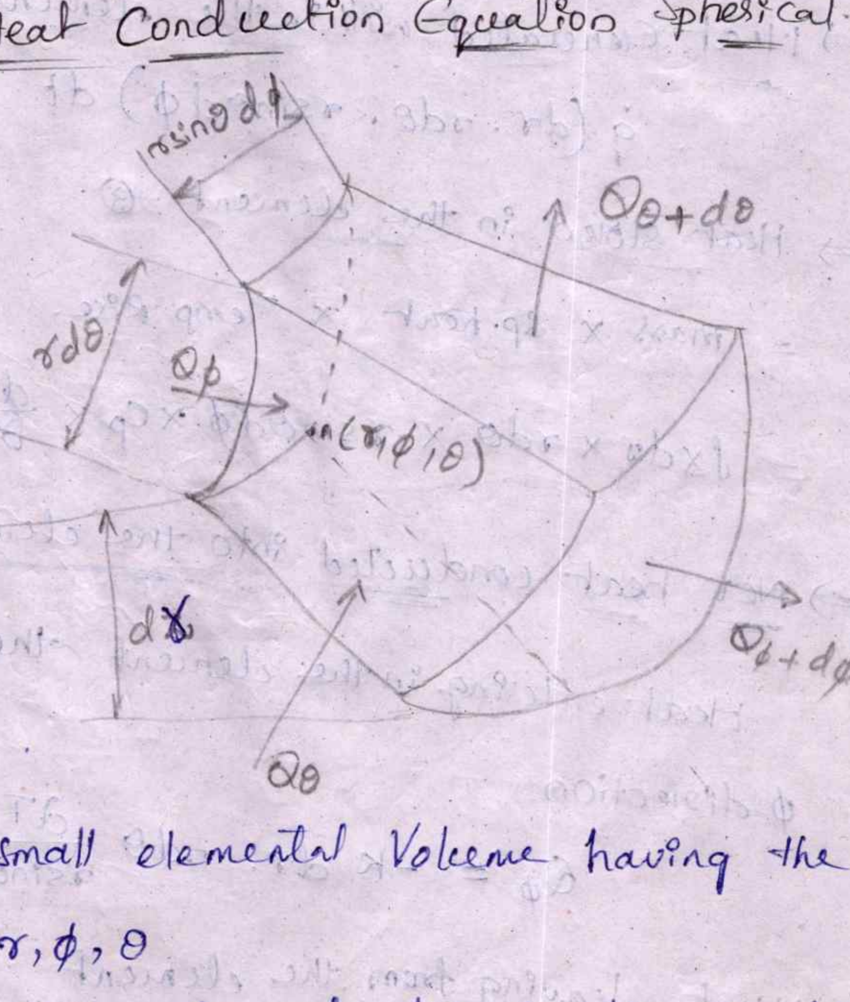
$= Q_\phi - Q_{\phi + d\phi}$

$= K\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}\right]\left[r d\phi \; dr \, dz\right) d\phi$

~~Adding equation.~~

~~$K(dr\ r d\phi$~~

General Heat Conduction Equation Spherical Coordinate



Consider a small elemental Volume having the co-ordinates r, ϕ, θ

$\Rightarrow$ The volume of the element $dV = dr\, r d\theta\; r\sin\theta\, d\phi$

Let us ~~consider~~ Assume that thermal conductivity

$k, \rho, \& C_p$ = constant

⇒ The energy balance of this sperical element is obtained from first law of Thermodynamics.

Net heat conducted into the element from all co-ordinate directions.	+	Heat generated within the element	=	Heat stored in the element
①		②		③

⇒ Heat Generated within the element = ②

$$\dot{q}\,(dr \cdot r d\theta \cdot r\sin\theta\, d\phi)\, dt$$

⇒ Heat stored in the element - ③

= mass × sp. heat × Temp Rise.

$$= \rho \times dr \times r d\theta \times r\sin\theta d\phi \times C_p \times \frac{dT}{\partial t} dt$$

⇒ Net heat conducted into the element ⇒ ①

Heat entering in the element through r & θ plane.

ϕ direction

$$Q_\phi = -k\, dr \cdot r d\theta \frac{\partial T}{r\sin\theta d\phi} dt \longrightarrow (a)$$

⇒ Heat leaving from the element

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\sin\theta\,\partial\phi}(Q_\phi)\, r\sin\theta d\phi \quad - (b)$$

Net heat conducted in ϕ direction

$$d\phi = Q_\phi - Q_{\phi+d\phi}$$

$$= Q_\phi - Q_\phi - \frac{\partial}{r\sin\theta\,\partial\phi}(Q_\phi)\,r\sin\theta\,d\phi$$

$$= \frac{\partial}{r\sin\theta\,d\phi}\left(K\,dr\,.\,r\,d\theta\,\frac{\partial T}{r\sin\theta\,\partial\phi}\,\partial t\right) r\sin\theta\,d\phi$$

$$= K_\phi\,dr.\;r\,d\theta\,.\,r\sin\theta\,d\phi\,.\,\frac{1}{r^2\sin^2\theta}\,dt\,\frac{\partial^2 T}{\partial\phi^2}$$

⇒ Heat entering in the element through θ & ϕ plane in r-direction

$$Q_r = -K\;r\,d\theta.\,r\sin\theta\,d\phi\;\frac{\partial T}{\partial r}\,dt$$

Heat leaving

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r)\,dr$$

Net heat conducted into the element.

$$dQ_r = Q_r - Q_{r+dr}$$

$$= Q_r - Q_r - \frac{\partial}{\partial r}(Q_r)\,dr$$

$$= \frac{\partial}{\partial r}\left(K.\,r\,d\theta\;r\sin\theta\,d\phi\;\frac{\partial T}{\partial r}\,dt\right)dr$$

$$dQ_r = K\,dr.\,r\,d\theta.\,r\sin\theta\,d\phi\;dt \times \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

⇒ Heat entering into the element ϕ & r plane in θ direction.

⇒ Heat enetering

$$Q_\theta = -K \; dr. \; r\sin\theta \, d\phi \; \frac{\partial T}{r\partial\theta} \; dt$$

⇒ Heat leaving

$$Q_{\theta+d\theta} = Q_\theta + \frac{\partial}{r\partial\theta}(Q_\theta) \, r d\theta$$

⇒ Net heat condcected

$$dQ_\theta = Q_\theta - Q_{\theta+d\theta}$$

$$= \frac{\partial}{r\partial\theta}\left(K \, dr . \, r\sin\theta \, d\phi \; \frac{\partial T}{r\partial\theta} \; dt\right) r d\theta$$

$$dQ_\theta = K \, dr. \, r d\theta. \, r\sin\theta \, d\phi \; dt \times \frac{1}{r^2 \sin\theta} \times \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right)$$

Net heat conducted into the element from all the co-ordinate direction.

$$= K \, dr. \, r d\theta . \, r\sin\theta \, d\phi \, dt \left[\frac{1}{r^2\sin^2\theta} \frac{\partial^2 T}{\partial\phi^2} + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right)\right]$$

$$= K . \left(\frac{1}{r^2 \sin^2\theta} . \frac{\partial^2 T}{\partial\phi^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right)\right) + q = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}$$

$$= \frac{1}{r^2 \sin^2\theta} . \frac{\partial^2 T}{\partial\phi^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial T}{\partial\theta}\right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

2nd derivation
~~Heat Entering in the element~~

⇒ heat entering into the element through ϕ & z plane in r-direction within the time $d\theta$.

$$Q_r = -K\, r\, d\phi\, dz\, \frac{\partial T}{\partial r} d\theta.$$

⇒ heat leaving

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r) \cdot dr$$

⇒ Net heat conducted

$$dQ = Q_r - Q_{r+dr}$$

$$= -\frac{\partial}{\partial r}(Q_r) \cdot dr$$

$$= -\frac{\partial}{\partial r}\left(K r\, d\phi\, dz \cdot \frac{\partial T}{\partial r} d\theta\right) dr$$

$$= K \cdot d\phi\, dz\, d\theta\, dr \left(\frac{\partial}{\partial r}\left(r \cdot \frac{\partial T}{\partial r}\right)\right)$$

$$= K \cdot \partial\phi\, \partial z\, d\theta\, dr \left[\frac{\partial T}{\partial r} + r \cdot \frac{\partial^2 T}{\partial r^2}\right]$$

$$= k\, r\, d\phi\, dr\, dz\, d\theta \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right]$$

⇒ Heat Entering into the ϕ direction through z & r direction in time $d\theta$.

$$Q_\phi = -k\, dz\, dr \cdot \frac{\partial T}{r\,\partial\phi}\, d\theta.$$

⇒ Heat leaving

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\,\partial\phi}(Q_\phi)\, r\, d\phi$$

$\Rightarrow$ Net heat conducted

$$dQ_\phi = Q_\phi - Q_{\phi+d\phi}$$

$$= -\frac{\partial}{r\partial\phi}(Q_\phi)\, r\,d\phi$$

$$= \frac{\partial}{r\partial\phi}\left(k\,dz\,dr\,\frac{\partial T}{r\partial\phi}\,d\theta\right) r\,d\phi$$

$$= k(dz\,dr.\,r\,d\phi)\,d\theta \times \frac{1}{r^2} \times \frac{\partial^2 T}{\partial\phi^2}$$

$$= k(dr.\,r\,d\phi.\,dz).\,\frac{1}{r^2}.\frac{\partial^2 T}{\partial\phi^2}.\,d\theta.$$

$$k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2}\right) dr\, r\,d\phi\,dz\, d\theta$$

$$+ \dot{q}(dr\,r\,d\phi\,dz)\,d\theta = \rho(dr\,r\,d\phi\,dz)\,c_p\frac{\partial T}{\partial\theta}\,d\theta$$

$$\boxed{\frac{\partial^2 T}{\partial r^2}} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2}\Big) + \frac{\dot{q}}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial\theta}$$

$$= \frac{1}{\alpha}\frac{\partial T}{\partial\theta}$$

20/02/2023

UNIT - 2

If the temperature of a body doesnot Varying with time, it is said to be in a study state and that type of condition is known as study state condiction.

If the temperature of a body varies with time, it is said to be in a transient state and that type of condition is known as transient condition or study state condition

Transient heat condition can be divided into p.

1) Periodic
2) Non Periodic Heat flow.

In Periodic heat flow, the temp varies on a regular basis.

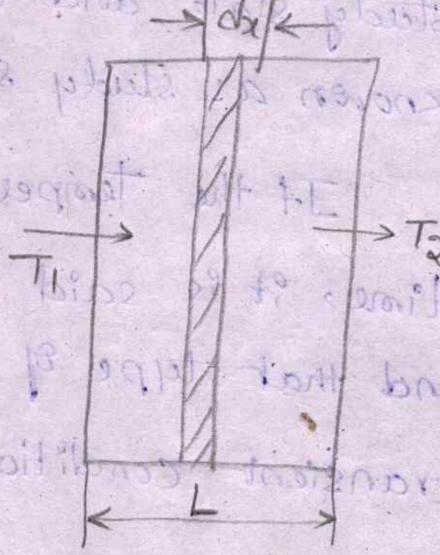
ex: Surface of earth during a period of 24hrs.

In Non-Periodic heat flow, the temp at any point within the system Varies non linearly with time

ex: Cooling of bars

Conduction of heat through a Slab or Plain wall

Consider a slab of uniform Thermal Conductivity (K), thickness (L), with inner Temperature (T_1) and outer temperature (T_2).



$$Q = -KA\frac{dT}{dx}$$

$$Q.dx = -KA\,dT$$

Integrating the above eqn

$$Q.\int_0^L dx = -KA\int_{T_1}^{T_2} dT$$

$$Q.[x]_0^L = -KA[T]_{T_1}^{T_2}$$

$$Q.L = -KA[T_2 - T_1]$$

$$Q.L = KA[T_1 - T_2]$$

$$\left[\because \frac{L}{KA} = R\right]$$

thermal resistance

$$Q = \frac{KA[T_1 - T_2]}{L}$$

$$= \frac{T_1 - T_2}{\frac{L}{KA}}$$

$$\boxed{Q = \frac{\Delta T_{overall}}{R}}$$

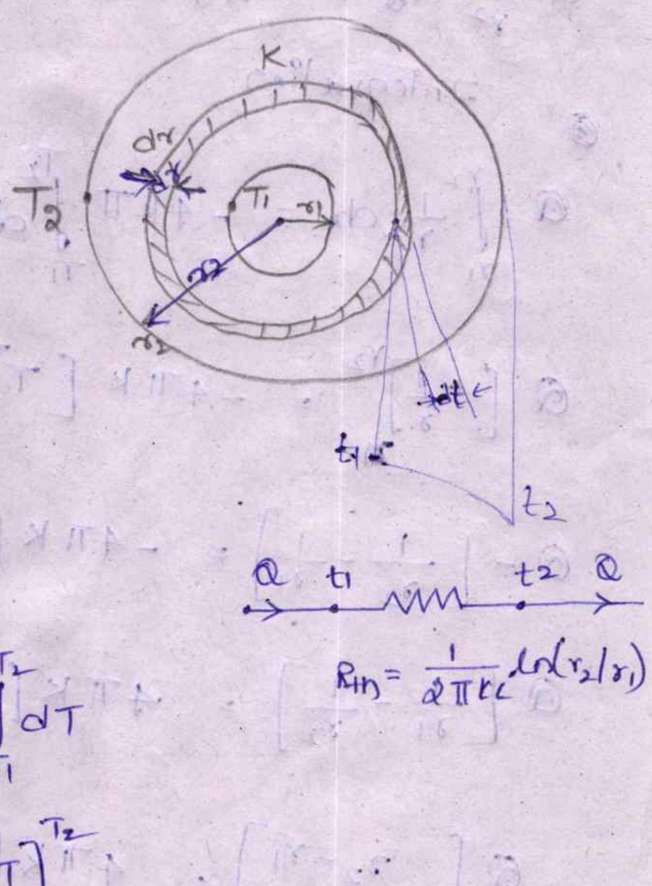
Conduction of heat through ~~sph~~ Hallow Cylinder

Consider a hallow cylinder of inner radius(r_1), outer radius (r_2), inner temp(T_1), outer temp (T_2) and thermal conductivity (K).

⇒ Let us consider a small elemental area of thickness dr.

⇒ From Fourier law of conduction, we know that

~~sphere~~ Area $= 2\pi r L$



$$Q = -KA \frac{dT}{dx}$$

$$Q = -K\,2\pi r L \frac{dT}{dr}$$

$$Q \cdot \frac{1}{r} dr = -K\,2\pi L\, dT$$

$R_{th} = \frac{1}{2\pi KL} \ln(r_2/r_1)$

Integrating

$$Q \cdot \int_{r_1}^{r_2} \frac{1}{r} dr = -K\,2\pi L \int_{T_1}^{T_2} dT$$

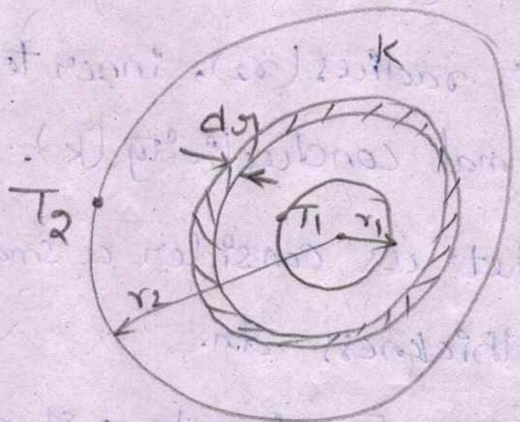
$$Q \ln[r]_{r_1}^{r_2} = -2\pi KL\,[T]_{T_1}^{T_2}$$

$$Q \cdot \ln[r_2 - r_1] = -2\pi KL\,(T_2 - T_1)$$

$$Q \cdot \ln\left[\frac{r_2}{r_1}\right] = 2\pi KL\,(T_1 - T_2)$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi KL} \cdot \ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T_{overall}}{R}$$

Conduction of heat through hallow Sphere



$$A = 4\pi r^2$$

$$Q = -KA\frac{dT}{dx}$$

$$Q = -K \cdot 4\pi r^2 \cdot \frac{dT}{dr}$$

$$Q \cdot \frac{1}{r^2}\, dr = -K\, 4\pi\, dT$$

Integrating

$$Q \int_{r_1}^{r_2} \frac{1}{r^2}\, dx = -4K\pi \int_{T_1}^{T_2} dT$$

$$Q\left[-\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi K\left[T\right]_{T_1}^{T_2}$$

$$Q - \left[\frac{1}{r_2} - \frac{1}{r_1}\right] = -4\pi K\left[T_2 - T_1\right]$$

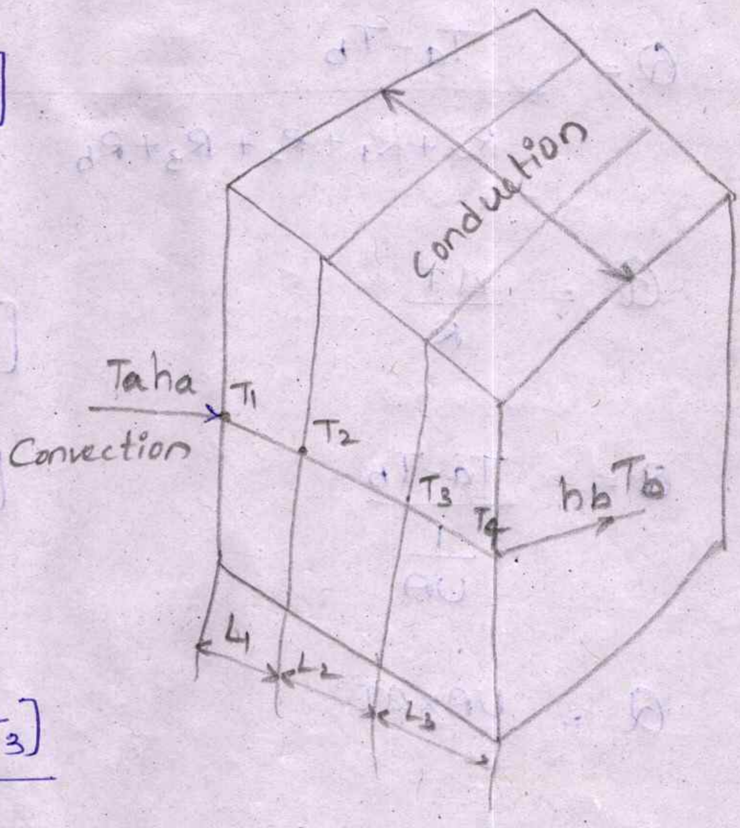
$$Q\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi K\left[T_1 - T_2\right]$$

$$Q\left[\frac{r_2 - r_1}{r_1 r_2}\right] = 4\pi K\left[T_1 - T_2\right]$$

$$Q = \frac{T_1 - T_2}{\frac{1}{4\pi K}\left(\frac{r_2 - r_1}{r_1 r_2}\right)}$$

$$Q = \frac{\Delta T_{overall}}{R}$$

Heat transfer through a composite wall with inside and outside Convection



$$\rightarrow Q = h_a A \left[T_a - T_1 \right]$$

$$T_a - T_1 = Q \times \frac{1}{h_a A}$$

$$\rightarrow Q = \frac{K_1 A \left[T_1 - T_2 \right]}{L_1}$$

$$T_1 - T_2 = Q \times \frac{L_1}{K_1 A}$$

$$\rightarrow Q = \frac{K_2 A \left[T_2 - T_3 \right]}{L_2}$$

$$T_2 - T_3 = \frac{Q \times L_2}{K_2 A}$$

$$\rightarrow Q = \frac{K_3 A \left[T_3 - T_4 \right]}{L_3}$$

$$T_3 - T_4 = \frac{Q \times L_3}{K_3 A}$$

$$\rightarrow Q = h_b A \left[T_4 - T_b \right]$$

$$T_4 - T_b = Q \times \frac{1}{h_b A}$$

$$T_a - T_b = Q\left[\overset{R_a}{\frac{1}{h_a A}} + \overset{R_1}{\frac{L_1}{k_1 A}} + \overset{R_2}{\frac{L_2}{k_2 A}} + \overset{R_3}{\frac{L_3}{k_3 A}} + \overset{R_b}{\frac{1}{h_b A}}\right]$$

$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_3 + R_b}$$

$$Q = \frac{\Delta T}{R} \qquad \left[\because R = \frac{1}{UA}\right]$$

$$Q = \frac{T_a - T_b}{\frac{1}{UA}} \qquad \left[\because \Delta T = T_a - T_b\right]$$

$$Q = UA \times \Delta T$$

① Sol

Given data

length = 6 m

height = 4 m

thickness L = 0.30 m

A = Length x height
= 6x4
= 24

$T_1 = 100°C + 273 = 373 K$

$T_2 = 40°C + 273 = 313 K$

$K = 0.55$ w/mK

$$Q = \frac{\Delta T}{R} = \frac{(T_1 - T_2)}{R}$$

$$R = \frac{L}{KA} = \frac{0.3}{0.55 \times 24} = 0.0227$$

$$Q = \frac{(373-313)}{0.0227} = 2643 \text{ W}$$

② Sol

Given that

$T_a = -20°C + 273 = 253 K$

$T_b = 25°C + 273 = 298 K$

$L_3 = L_1 = 30 cm = 0.3$

$L_2 = 0.2 m$.

$K_3 = K_1$ for brick $= 2.3$ w/mK

cork — $K_2 = 0.05$ w/mK

{ from data book 15 Pg.

$h_b = 55.4$ w/m²K

$h_a = 17$ w/m²K

$$\frac{Q}{A} = \frac{\Delta T}{R} =$$

$$Q = \frac{\Delta T}{R} = \frac{298-253}{4.337} = 10.37 \text{ W}$$

$$R = \left[\frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_b}\right] = 4.337$$

(3)
Sol

$L = 250mm = 0.25m$

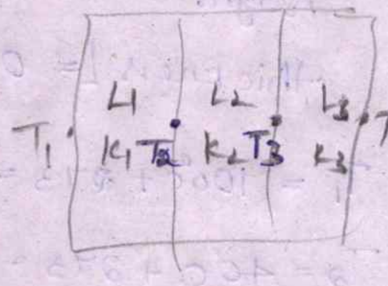
$K = 1.05 \, W/mK.$

insulation brick $L_2 = 0.12m.$

$K_2 = 0.15 \, W/mK$

Red brick $L_3 = 200mm = 0.2m$

$K_3 = 0.85 \, W/mK$



$T_1 = 850+273 = 1123 K$

$T_4 = 65+273 = 338 K$

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{A}\left[\cancel{\frac{1}{h_a}} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \cancel{\frac{1}{h_b}}\right]$$

$$\frac{Q}{A} = \frac{T_1 - T_4}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}} = 616.46 \, W/m^2$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{L_1/K_1} = \frac{T_2 - T_3}{\frac{L_2}{K_2}} = \frac{T_3 - T_4}{\frac{L_3}{K_3}}$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{L_1/K_1} = 976.22 K = T_2$$

$T_3 = 483.05 K.$

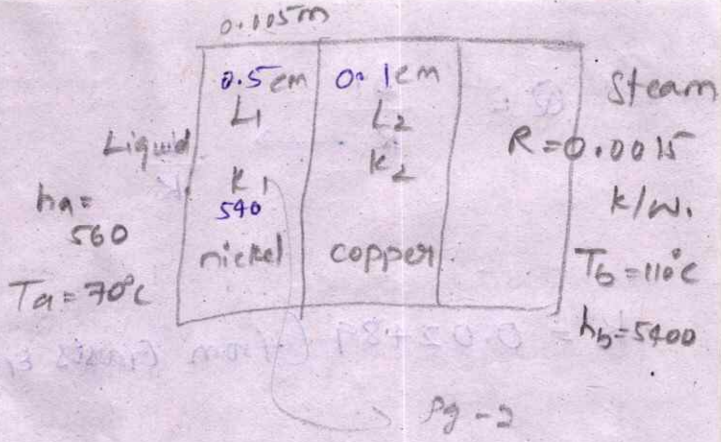
(or) $616.46 = \dfrac{T_3 - 338}{\frac{0.2}{0.85}}$

$T_3 = 483.05K.$

④ sol

Given data

$A = 25.2\, m^2$



$$R = \frac{1}{UA}$$

$$R = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \overset{R}{\left(\frac{L_3}{k_3}\right)} + \frac{1}{h_b}\right]$$

$$= \frac{1}{25.2}\left[\frac{1}{560} + \frac{0.005}{90} + \frac{0.001}{386} + 0.0015 + \frac{1}{5400}\right]$$

$R = 1.4 \times 10^{-4}\ k/w.$

$$Q = \frac{\Delta T}{R} = \frac{(343 - 383)}{1.4 \times 10^{-4}} = -28.3 \times 10^{4}\ W$$

ⓐ $Q = UA\,\Delta T$

$-28.3 \times 10^{4} = U \times 25.2 \times (343 - 383)$

$U = -280.75\ w/m^2 k$

~~Scaling~~

ⓑ $\Rightarrow Q = \frac{\Delta T}{R}$

$$28.3 \times 10^{4} = \frac{\Delta T}{0.0015}$$

$\Delta T = 424.5\, K$

(or) $151.5\,°c$

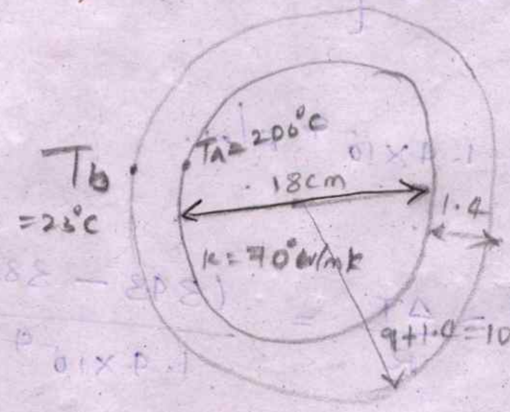
$$Q = \frac{\Delta T}{R} \longrightarrow \frac{L}{K}$$

$K = 0.02489$ (from Gases & Vapour Pg. 48) at 110°c

$$28.3 \times 10^4 = \frac{424.5}{\frac{L}{0.02489}}$$

$L = 3.7 \times 10^{-5}$ m

(6)
Sol



$$Q = \frac{T_a - T_b}{R}$$

$$R = \frac{1}{2\pi \cancel{L}}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} \ln\left[\frac{r_2}{r_1}\right]\right] \longrightarrow \text{Pg.54 composite cylinder}$$

$$R_L = \frac{1}{2\pi}\left[\frac{1}{690 \times 0.9} + \frac{1}{70} \ln\left[\frac{10.4 \times 10^{-2}}{9 \times 10^{-2}}\right]\right]$$

$R = 2.8 \times 10^{-3}$ K/W

$$Q = \frac{T_a - T_b}{R} = \frac{(200+273) - (170+273)}{2.8 \times 10^{-3}} = 10.7 \times 10^3 \text{ W}$$

$Q = UA\Delta T$ $\left[\because Q = \frac{\Delta T}{R} = \frac{T_a - T_b}{R}\right]$

$= U \times 2\pi L r_2 \times \Delta T$

$\frac{Q}{L} = U \times 2\pi \times r_2 \times (T_a - T_b)$

$10.7\times10^3 = U \times 2\pi \times (10.4\times10^{-2}) \times \left(473 - \underbrace{(23+273)}_{296}\right)$

$U = 92.51$

(7)

Sol $K_1 = 65\ w/mK$

$r_1 = 60mm = 0.06\ m$

$r_2 = 175\ mm = 0.175\ m.$

thickness $= 10mm = 0.01$

insulation $K_2 = 10\ w/mK$, $r_3 = 175 + 10 = 185$

$T_a = 500 + 273 = 773\ K$

$T_b = 50^\circ + 273 = 323\ K$

→ Composite sphere with Convection → Pg: 54

$Q = \frac{\Delta T}{R}$

$R = \frac{1}{4\pi}\left[\cancel{\frac{1}{h_i r_1^2}} + \frac{1}{K_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + \frac{1}{K_2}\left[\frac{1}{r_2} - \frac{1}{r_3}\right] + \cancel{\frac{1}{h_o r_3^2}}\right]$

→ neglecting h_i and h_o,

$R = \frac{1}{4\pi}\left[\frac{1}{65}\left[\frac{1}{0.06} - \frac{1}{0.175}\right] + \frac{1}{10}\left[\frac{1}{0.175} - \frac{1}{0.185}\right]\right]$

$= 0.01586$

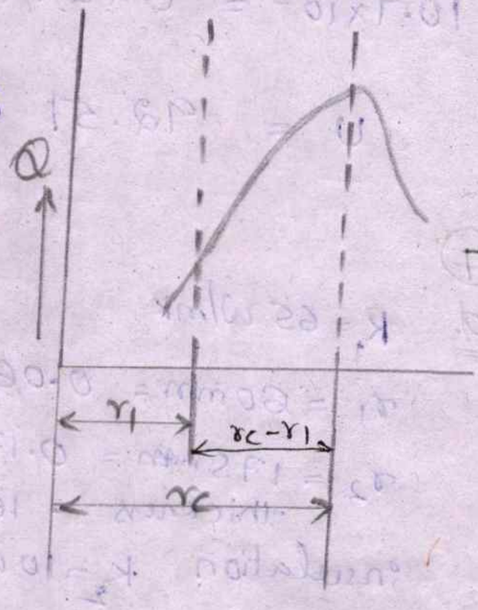
$Q = \frac{T_a - T_b}{R} = \frac{773 - 323}{0.01586} = 28373.26\ W$ (or) $28.3\times10^3\ W$

* Critical Radius of Insulation

Addition of Insulating Material on a surface doesnot reduce the amount of heat transfer rate always.

r_c = critical radius

$r_c - r_1$ = critical thickness



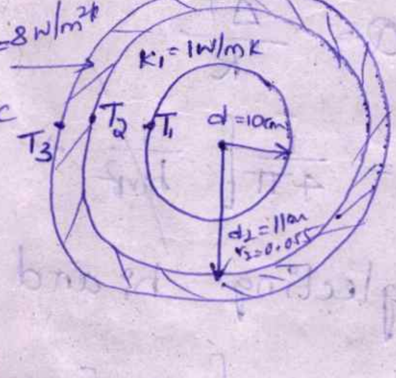
$$Q = \frac{T_i - T_\infty}{\frac{\ln\left(\frac{r_0}{r_1}\right)}{2\pi K L}}$$

$$Q = \frac{T_i - T_\infty}{\frac{\ln\left(\frac{r_0}{r_1}\right)}{2\pi K L} + \frac{1}{A_0 h}} \qquad \left[\because A_0 = 2\pi r_0 \cancel{h}\right]$$

$$r_0 = \frac{K}{h} = r_c$$

(8)

Sol. Given data



$$r_c = \frac{k}{h} = \frac{1}{8} = 0.125m$$

$$Q = \frac{\Delta T}{R} = \frac{T_a - T_b}{R}$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \cancel{\frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right)} + \cancel{\frac{1}{k_3}\ln\left(\frac{r_4}{r_3}\right)} + \frac{1}{h_b r_4}\right]$$

→ Pg. 54

$$= \frac{1}{2\pi L}\left[\frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_b r_c}\right]$$

$$\frac{1}{2\pi L} \times \left[\frac{1}{1}\ln\left(\frac{0.055}{0.05}\right) + \frac{1}{8 \times 0.125}\right]$$

$$\frac{R}{1} = \frac{0.174}{L}$$

$$Q = \frac{T_a - T_b}{\frac{0.174}{L}}$$

Take $L = 1\,m$.

$$\frac{Q}{L} = \frac{473 - 293}{0.174} = 1034 \text{ W/m}.$$

→ outer surface Temperature.

$$\frac{Q}{L} = \frac{T_a - T_b}{R} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_L} = \frac{T_3 - T_b}{R_3}$$

$$\frac{Q}{L} = \frac{T_3 - T_b}{R_3} \rightarrow \boxed{R_3 = \frac{1}{2\pi \cancel{K} L} \times \frac{1}{h_b \times r_c}}$$

$$1034 = \frac{T_3 - 293}{\frac{1}{2\pi} \times \frac{1}{8 \times 0.125}}$$

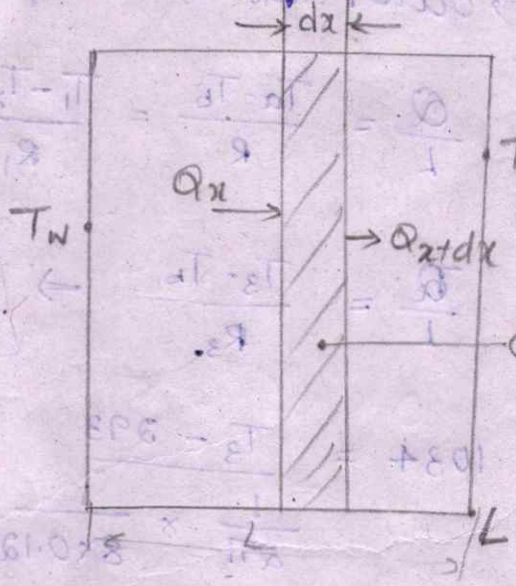
$$T_3 = 457.56\,K$$

* Heat Conduction with Heat Generation

In many practical cases, there is a heat generation within the system. typical examples
1) electrical coil 2) resistance heater
3) Nuclear reactor
4) combustion of fuel in fuel bed of the boiler furnance

Plane Wall with Internal Heat Generation

Consider a slab of thickness L, thermal conductivity K as shown in fig.
⇒ consider a small elemental area of thickness (dx)



from fourier Law of equation, we know that

⇒ Heat transfer at x

$$Q_x = -KA\frac{dT}{dx}.$$

⇒ Heat conducted out at $Q_{x+dx} = Q_x + \frac{d}{dx}(Q_x)dx$

⇒ Heat generated within dx

$$Q_g = \dot{q}\,A\,dx$$

We know that

$$Q_x + Q_g = Q_{x+dx}$$

$$\cancel{Q_x} + Q_g = \cancel{Q_x} + \frac{d}{dx}(Q_x)\,dx$$

$$Q_g = \frac{d}{dx}\left(-KA\frac{dT}{dx}\right)dx$$

$$KA\frac{d^2T}{dx^2}\,dx + \dot{q}\,A\,dx = 0$$

$$\int \frac{d^2T}{dx^2} + \int \frac{\dot{q}}{K} = 0$$

$$\int \frac{dT}{dx} + \frac{\dot{q}}{K}\int x = \int C_1 \qquad \int x^n = \frac{x^{n+1}}{n+1}$$

$$T + \frac{\dot{q}}{K}\cdot\frac{x^2}{2} = C_1x + C_2$$

$$T = C_1x + C_2 - \frac{\dot{q}}{K}\cdot\frac{x^2}{2} \longrightarrow (a)$$

The temperature on the two faces of the wall (T_w) is the same because it looses the same amount of heat by convection on both sides then applying the boundary condition. $C_1 = 0$

$$C_1 = 0 \longrightarrow (1)$$

$$T = C_2 - \frac{\dot{q}}{K}\frac{x^2}{2}$$

Apply $T = T_w$, $x = \frac{L}{2}$

$$T_w = C_2 - \frac{\dot{q}}{K} \times \frac{1}{2} \times \frac{L^2}{4}$$

$$C_2 = T_w + \frac{\dot{q}L^2}{8K} \longrightarrow (2)$$

Substitute eqn (1) & (2) in eq (a), we get

$$T = 0 + T_w + \frac{\dot{q}L^2}{8K} - \frac{q}{k}\frac{x^2}{2}$$

$$T = T_w + \frac{\dot{q}}{8K}(L^2 - 4x^2)$$

⇒ The maximum temperature T_{max} (at the Centre is obtained by putting at $x = 0$ in above eqn

$$T_{max} = T_w + \frac{\dot{q}L^2}{8K}$$

Heat flow rate

$$Q = \frac{1}{2}\dot{q}AL$$

Heat transfer by convection.

$$Q = hA(T_w - T_\infty)$$

$$\frac{1}{2}qAL = hA(T_w - T_\infty)$$

$$\frac{1}{2}\dot{q}AL = hAT_w - hAT_\infty$$

$$h\cancel{A}T_w = \frac{1}{2}q\cancel{A}L + h\cancel{A}T_\infty$$

$$hT_w = \frac{1}{2}qL + hT_\infty$$

$$T_w = T_\infty + \frac{\dot{q}L}{2h}$$

Cylinder with Internal Heat Generation

Consider a cylinder of radius ~~small~~ (r) and thermal conductivity (k).

Heat is generated Q_g

$\Rightarrow$ In the cylinder due to passage of an electric current. from the fourier law of conduction we know that 1-Dimensional cylindrical coordinate with heat generation is

$$\frac{r d^2T}{dr^2} + \frac{\dot{q} r}{k} = 0$$

Integrating the above equation

$$r\int \frac{d^2T}{dr^2} + \int \frac{\dot{q} r}{k} = \int 0$$

$$r\frac{dT}{dr} + \frac{\dot{q}}{k} \cdot \frac{r^2}{2} = C_1$$

$$\frac{dT}{dr} + \frac{\dot{q} r}{2k} = \frac{C_1}{r}$$

Integrating the above equation

$$T + \frac{\dot{q}}{2k} \frac{r^2}{2} = C_1 \ln r + C_2$$

$$T = C_1 \ln r + C_2 - \frac{\dot{q} r^2}{4k} \longrightarrow (1)$$

Applying boundary condition $C_1 = 0$, $r = r_0$, $T = T_w$

$$T_w = C_2 - \frac{\dot{q} r_0^2}{4K}$$

$$C_2 = T_w + \frac{\dot{q} r_0^2}{4K}$$

Applying the C_1 & C_2 equations in eq ①

$$T = T_w + \frac{\dot{q} r_0^2}{4K} - \frac{\dot{q} r^2}{4K}$$

$$T = T_w + \frac{\dot{q}}{4K}\left(r_0^2 - r^2\right) \longrightarrow ②$$

At centre $r = 0$, $T = T_{max}$

$$T_{max} = T_w + \frac{\dot{q}}{4K}\left(r_0^2\right)$$

Heat generated

$$Q = \pi r_0^2 L \dot{q} \longrightarrow ⓐ$$

Heat transfer due to convection

$$Q = hA(T_w - T_\infty)$$

$$= h\, 2\pi r_0 L\,(T_w - T_\infty) \longrightarrow ⓑ$$

$a = b$ then

$$T_w = T_\infty + \frac{r_0 \dot{q}}{2h}$$

① An electric current is passed through a plane of thickness 150mm, which generate heat at the rate of 50,000 W/m³. The convective heat transfer coefficient b/w wall and ambient are 65 W/m²K, ambient air temp is 28°C and thermal conductivity of wall is 22 W/mK. Calculate (i) surface temperature

(ii) maximum temp in the wall

Sol: Wall thickness $L = 150mm$
$= 0.15 m$

[from study state conduction with heat generation 56 Pg]

$\dot{q} = 50,000 \ W/m^3$

$h = 65 \ W/m^2K$.

air temp $T_\infty = 28°C + 273 = 301 \ K$.

thermal conductivity $k = 22 \ W/mK$

(i) Surface temp (slab)

$$T_w = T_\infty + \frac{qL}{h}$$

$$= 301 + \frac{50,000 \times 0.15}{65}$$

$$\boxed{T_w = 416.3 K}$$

(ii) Max. temp

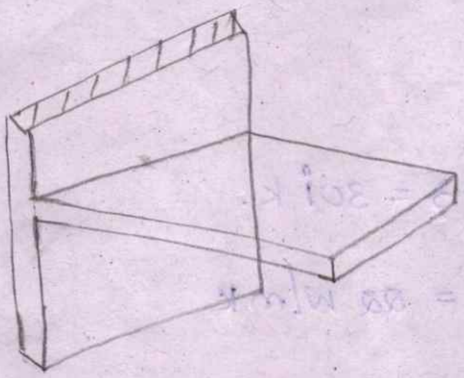
$$T_0 = T_w + \frac{q}{2k} L^2$$

$$= 416.3 + \frac{50,000}{2 \times 22} \times 0.15^2$$

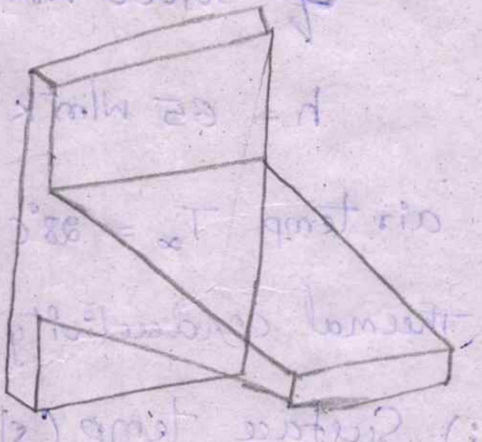
$$\boxed{T_0 = 441.36\ k}$$

Fins :

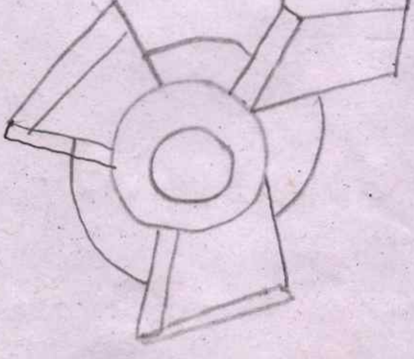
It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surface used for increasing heat transfer are called extended surfaces are called fins.



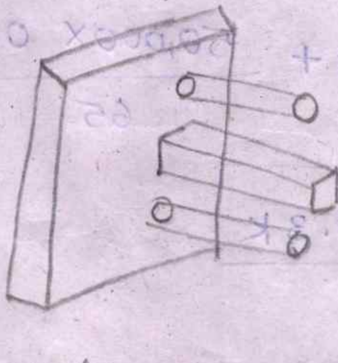
uniform straight fin



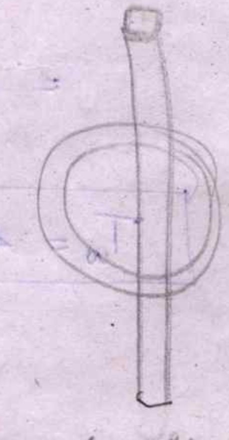
Tapered straight fin



Splines



Pin fin



Annular fin

Commonly there are three types of fins

1) Infinitely long fin
2) short fin (end is insulated)
3) short fin (end is not insulated)

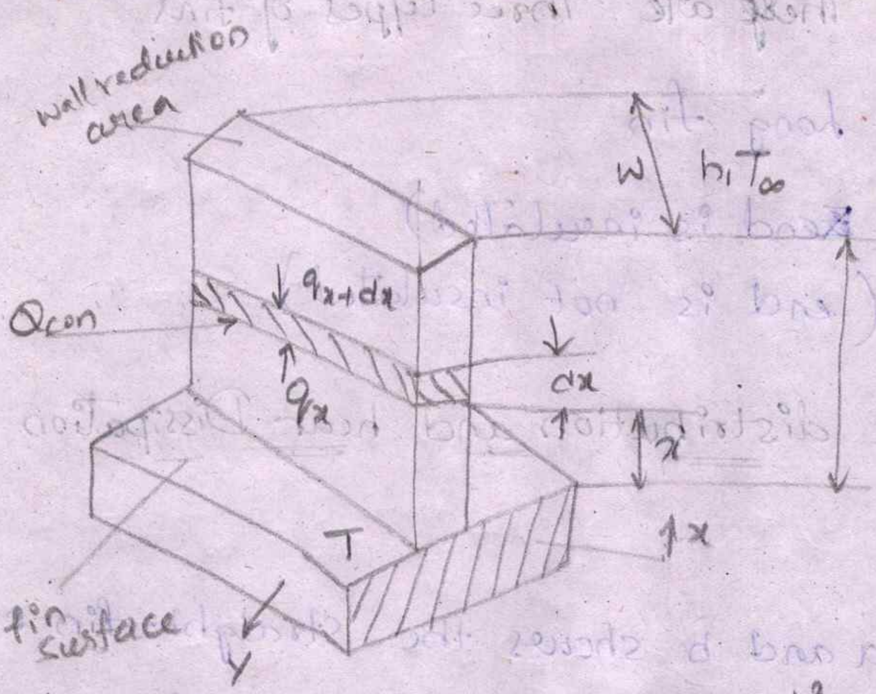
Temperature distribution and heat Dissipation in Fin

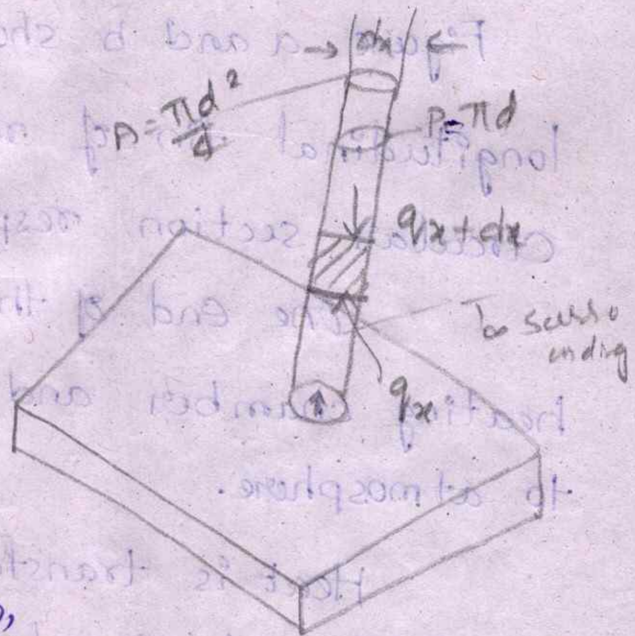
Figure a and b shows the straight fin or longitudinal fin of rectangular section and circular section respectively.

One end of the fin is enclosed in a heating chamber and the other end is exposed to atmosphere.

Heat is transfered across the rectangu-lar field and circular rod by conduction. from the surface of fins, heat is transfered to a by convection.

Let us consider a small elemental area of thickness dx, which at a distance of x from the base.





A study state condition,

Heat balance equation for that element is as follows.

$$\left\{\begin{matrix}\text{Heat}\\ \text{conducted}\\ \text{into the}\\ \text{element}\end{matrix}\right\} = \left\{\begin{matrix}\text{Heat}\\ \text{condeected}\\ \text{out of the}\\ \text{element}\end{matrix}\right\} + \left\{\begin{matrix}\text{Heat convecting}\\ \text{to the}\\ \text{sursocending}\\ \text{air}\end{matrix}\right\}$$

$$-Q_x = Q_{x+dx} + Q_{convection}$$

$$-KA\frac{dT}{dx} = Q_x + \frac{d}{dx}(Q_x)dx + hA\Delta T$$

$$+ hA(T - T_\infty)$$

$$-KA\frac{dT}{dx} = Q_x + \frac{d}{dx}(Q_x)dx + hP.dx\,(T-T_\infty)$$

$$\cancel{Q_x} = \cancel{Q_x} + \frac{d}{dx}(Q_x)dx + hP.dx\,(T-T_\infty)$$

$$\frac{d}{dx}\left(KA\frac{dT}{dx}\right)\cancel{dx} = hP.\cancel{dx}\,(T-T_\infty)$$

$$\frac{d^2T}{dx^2}KA = hP(T-T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T-T_\infty) = 0$$

$$m^2 = \frac{hP}{KA} \qquad (\&)\quad \theta = T - T_\infty$$

$$m = \sqrt{\frac{hP}{KA}}$$

$$\theta = C_1 e^{-mx} + C_2 e^{+mx} \longrightarrow (1)$$

The abone equation shows that the is a function of x & it is a second order, linear differencial equation. $\frac{d^2T}{dx^2} + m^2\theta = 0$

The temperature distribution & heat dissipation depends upon the following condition.

Case (i) :- Infinitely long fin

In a fin is infinitely long, the temperature at its end is equally to that of its surrouding fluid (air)

$\Rightarrow$ at $x=0$, $T = T_b$

and x = infinity, $T = T_\infty$

$x = \infty$, $T = T_\infty$

$$\theta = c_1 e^{-mx} + c_2 e^{mx}.$$

$$\theta = T - T_\infty$$

Substitute $x=0, T = T_b$

$$T - T_\infty = c_1 e^{-mx} + c_2 e^{mx} \longrightarrow (2)$$

$$T_b - T_\infty = c_1 + c_2 \longrightarrow (3)$$

then substitute $x = \infty, T = T_\infty$

$$\cancel{T_\infty} - \cancel{T_\infty} = c_1 e^{-m\infty} + c_2 e^{m\infty}$$

$$0 = c_1 e^{-m\infty} + c_2 e^{m\infty} \longrightarrow (4)$$

substitute $c_2 = 0$ in eq (3), we get

$$T_b - T_\infty = c_1$$

Substitute c_1 in eq (2)

$$T - T_\infty = (T_b - T_\infty) e^{-mx} + 0 \qquad \left[\because m = \sqrt{\frac{hP}{kA}}\right]$$

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \Longrightarrow T - T_\infty = e^{-mx}(T_b - T_\infty)$$

Heat dissipation through the pin is obtained by integrating the heat lost by convection over the entire fin surface, we know that

$\Rightarrow$ Heat lost by convection

$$Q = hA(T - T_\infty)$$

$$= hP.dx(T - T_\infty) \qquad \left[\because e^{\infty} = -1\right]$$

$$= \int_0^\infty hP.e^{-mx}(T_b - T_\infty)\,dx$$

$$= hP(T_b - T_\infty)\int_0^\infty e^{-mx}\,dx.$$

$$= hP(T_b - T_\infty) \times \frac{-1}{m}\left[e^{-mx}\right]_0^\infty$$

$$= hP(T_b - T_\infty) \times \frac{-1}{m}\left[e^{0} - e^{-\infty}\right]$$

$$= hP(T_b - T_\infty) \times \frac{-1}{m}(-1)$$

$$= hP(T_b - T_\infty \times \frac{1}{m}$$

$$= hP(T_b - T_\infty) \times \frac{1}{\sqrt{\frac{hP}{KA}}}$$

$$= hP(T_b - T_\infty) \times \sqrt{\frac{KA}{hP}}$$

$$= (hP)^{\cancel{1}\frac{1}{2}+\frac{1}{2}} (T_b - T_\infty) \times \frac{\sqrt{KA}}{\cancel{\sqrt{hP}}}$$

$$= \sqrt{hPKA}$$

$$Q = \sqrt{hPKA} \times (T_b - T_\infty)$$

Case (ii) : fin with insulated end (short fin)

The fin has a finite length and the tip of fin is insulated, at $x=0$ & $T=T_b$ $\longrightarrow$ (b)

$x=L$; $\frac{dT}{dx}=0 \longrightarrow$ (a)

$(T-T_\infty) = C_1 e^{-mx} + C_2 e^{mx} \longrightarrow$ (i)

diff eq (i) w.r.t x

$\frac{dT}{dx} = C_1 e^{-mx}(-m) + C_2 e^{mx}(m) \longrightarrow$ (c)

Apply (a) condition in (c)

$0 = C_1 e^{-mL} \times -m + C_2 e^{mL} \times m$

$\cancel{m} C_1 e^{-mL} = \cancel{m} C_2 e^{mL}$

$C_1 = C_2 e^{2mL} \longrightarrow$ (d)

Apply (b) conditions in (i)

$T_b - T_\infty = C_1 e^0 + C_2 e^0 \longrightarrow$ (ii)

Substitute C_1 in eq (ii)

$T_b - T_\infty = C_2 e^{2mL} + C_2$

$$C_2 = \frac{T_b - T_\infty}{[e^{2mL}+1]} \longrightarrow (e)$$

Apply (e) in (d)

$$C_1 = \frac{T_b - T_\infty}{e^{2mL}+1} \times e^{2mL}$$

$e^{-mL} = e^{mL}$

$C_1 = \frac{e^{mL}}{e^{-mL}}$

$= e^{mL} \times e^{mL}$

$= e^{2mL}$

$\frac{1}{e^{2mL}+1} \times e^{2mL}$

$\frac{1}{(e^{2mL}+1)} \times \frac{1}{e^{-2mL}}$

$\frac{1}{e^{-2mL} \times (e^{2mL} \times e^{-2mL})}$

$\frac{1}{e^{-2mL} + e^{\cancel{2mL} - \cancel{2mL}}}$

$$C_1 = \frac{T_b - T_\infty}{1 + e^{-2mL}}$$

Substitute C_1 & C_2 in eq (i)

$$T - T_\infty = \frac{T_b - T_\infty}{1 + e^{-2mL}} \times e^{-mx} + \frac{T_b - T_\infty}{e^{2mL} + 1} \times e^{mx}$$

$$= T_b - T_\infty \left[\frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}} \right]$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \left[\frac{e^{-mx}}{1 + e^{-2mL}} \times \frac{e^{mL}}{e^{mL}} + \frac{e^{mx}}{1 + e^{2mL}} \times \frac{e^{-mL}}{e^{-mL}} \right]$$

$$= \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}}$$

Applications of fin

The main applications of fins

⇒ Cooling of electronical components

⇒ " motor cycle Engine

⇒ " small capacity compressors.

⇒ " transforms.

⇒ cooling of radiators & refrigerators etc.

Fin efficiency : It is the ratio of actual heat transfer of fin to the maximum possibility heat transfer by the fin.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max\,fin}} = 1$$

for insulated η efficiency $= \frac{\tanh(mL)}{mL}$

Fin effectiveness: It is defined as the ratio of heat transfer with fin to heat transfer without fin

$$\epsilon = \frac{Q_{with\ fin}}{Q_{without\ fin}} \quad \left[\because \frac{\epsilon_{mpla}}{\epsilon_{msal}} = \epsilon\right]$$

(9)

sol Given data

diamel $d = 3\,mm = 0.003\,m$.

base temp $T_b = 140^\circ C = 413\,K$

$K = 150\,w/mk$

$h = 300\,w/m^2k$

$T_\infty = 10^\circ C = 283\,K$

Heat transfer by fin $Q = (T_b - T_\infty)(hPKA)^{0.5}$

$\Rightarrow P =$ Perimeter $= (\pi d) = \pi \times 0.003 = 0.0094$

$A = \text{area} = \pi/4 \times d^2 = \frac{\pi}{4} \times (0.003)^2 = 7.06 \times 10^{-6}$

$Q = (413 - 283) \times (300 \times 0.0094 \times 150 \times 7.06 \times 10^{-6})^{0.5}$

$Q = 7.10\ W$

(10)

Sol

dia $d = 5cm = 0.05m$.

$T_b = 150°C = 423K$ $\quad h = ?$

$T_\infty = 20°C = 293K$

$k = 200\ W/mK$

$x = 20cm = 20 \times 10^{-2}\ m$

Intermediate Temp $T = 60°C = 333K$

$m = \sqrt{\frac{hP}{KA}}$

$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$

$\frac{333 - 293}{423 - 293} = e^{-m \times 20 \times 10^{-2}}$

$m = 5.89$

$\pi d = P$

$\pi \times 0.05 = 0.157$

$A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times 0.05^2$

$= 1.96 \times 10^{-3}\ m^2$

$\Rightarrow m = \sqrt{\frac{hP}{KA}} =$

$5.89 = \sqrt{\frac{h \times 0.157}{200 \times 1.96 \times 10^{-3}}} =$

$h = 86.6\ W/m^2 k$

(12)

sol

fin dimension = $0.5\ mm^2$ = $0.5 \times 10^{-3} m$ $\boxed{5 \times 10^{-7} m^2}$ (it is insulated)

long $l = 12mm = 12 \times 10^{-3} m$.

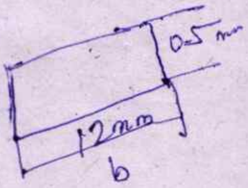
$T_b = 80°C = 353\ K$

$Q = 35 \times 10^{-3}\ W$

$k = 165\ W/mK$

$h = 10\ W/m^2K$

$T_\infty = 22 = 295\ K$



fin area $A = b \times t = 5 \times 5 \times 10^{-6} = 2.5 \times 10^{-7} m^2$

Perimeter $P = 2(b+t)$

$= 2(0.012 + 0.5 \times 10^{-3})$

$P = 2 \times 10^{-3} m$ (or) $0.025\ m$

$\Rightarrow$ no. of fins required(n) = $\dfrac{\text{heat generated}}{\text{heat transfer per fin}}$

$\Rightarrow$ short fin (end insulated) $Q = (hPkA)^{0.5}(T_b - T_\infty)\tanh(mL)$

$\Rightarrow m = \sqrt{\dfrac{hP}{kA}} = 22\ m^{-1}$

$\Rightarrow Q = 0.0135$ per fin

$n = \dfrac{35 \times 10^{-3}}{0.0135} = 2.59$

(14)
sol

wide $l = 140 mm = 0.014$

$t = 5mm = 0.005m.$

dia $d_1 = 200mm = 0.2m$

$r_1 = 100 mm = 0.1 m$

fin base temp $T_b = 170 = 443 K$

$T_\infty = 25°C = 298 K$

$K = 220 \, W/mK$

$h = 140 \, W/m^2K$

$L_c = L + \frac{t}{2} = 0.014 + \frac{0.005}{2}$

$= 0.1425 m$

$\Rightarrow r_{2c} = r_1 + L_c = 0.2425 m.$

$\Rightarrow A_m = t(r_{2c} - r_1) = 0.005(0.2425 - 0.1) = 7.125 \times 10^{-4} m^2$

$\Rightarrow A_s = 2\pi(r_{2c}^2 - r_1^2) = 2\pi(\quad) = 0.3065 m^2$

$x\text{-axis} = L_c^{1.5}\left(\frac{h}{KA_m}\right)^{0.5}$

$= 1.6$

$\text{curve} = \frac{r_{2c}}{r_1} = \frac{0.2425}{0.1} = 2.425$

$\eta = 35\%$

$Q = \eta A_s h (T_b - T_\infty)$

$Q = 2177 W$

Transient heat condition occurs in cooling of IC engines, Automobile engines, boiler tubes, heating & cooling of metal pillets, Rocket nozzles, electric iron etc.

Transient heat condition can be divided into

1) Periodic heat flow: Temperature varies on a regular basis.

ex: Cylinder of an IC engine.

2) Non-Periodic heat flow: Temperature at any point with in the system varies non linearly with time.

ex:- cooling of bars with atmospheric temperature and Ingote

Biot number: The ratio of Internal conduction resistance to the surface convection resistance. → 120 Pg.

$$Bi = \frac{hL}{k_s}$$

Significance

⇒ lumped parameter System $= \frac{hL}{k_s} < 0.1$ ↳ 66 Pg.

Bi = infinity (semi infinity solid)

→ Infinite solid $0.1 < B_i < 100$

⇒ ~~volume~~ =

Characteristic length:

$$L = \frac{V}{A}$$

V = area × thickness of the slab

A = 2 × area

$$L = \frac{\cancel{A} \times \text{thickness } (t)}{2 \times \cancel{A}}$$

$$L = t/2$$

for Cylinder

$$L = \frac{V}{A} = \frac{\pi R^2 \cancel{l}}{2\pi R \cancel{l}} = \frac{R}{2}$$ = Radius of cylinder.

for Sphere

$$L = \frac{V}{A} = \frac{\frac{4}{3}\cancel{\pi} R^{\cancel{3}}}{\cancel{4\pi} R^{\cancel{2}}} = \frac{R}{3}$$

Significance of Fourier Number: → 120Pg

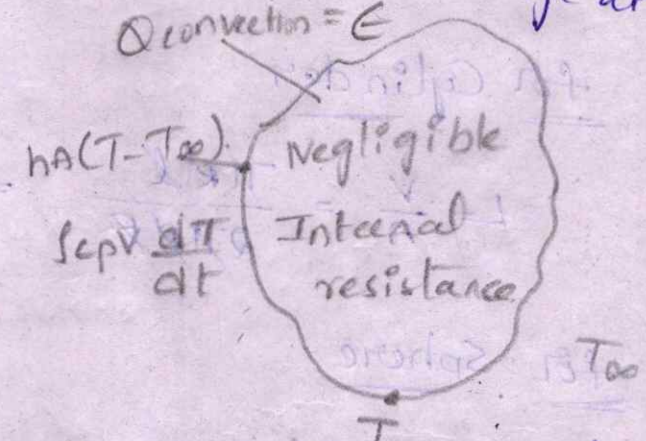
The Ratio of characteristic body dimension to the Temperature wave penetration depth in time, τ

Lumped Heat Analysis: (Negligible Internal Resistance)

The process in which the internal resistance is negligible in comparing with its surface Resistance in its Nutonian heating or cooling process :

In a newtons Nutonian heating or cooling process the temperature is considered to be uniform at a given time. such that Analysis is called as lumped parameter Analysis.

Let us consider a solid whose initial temperature is T_0 and it is placed suddenly in ambient air temperatur T_∞. The trancient response of the body can be determined by relating its rates of change of internal energy. with convective exchange at the surface



Convective Heat transfer = Rate of change of Internal Energy.

$$-hA(T-T_\infty) = \rho \times C_p \times V \times \frac{dT}{dt}$$

$$\frac{dT}{T-T_\infty} = \frac{-hA}{\rho C_p V} \times dt$$

$$\int \frac{dT}{T-T_\infty} = \frac{-hA}{\rho C_p V} \int dt$$

$$\ln\left[T-T_\infty\right] = \frac{-hA}{\rho C_p V} t + C_1 \longrightarrow (1)$$

Apply boundary conditions to eq (1), we get

At $T = T_0$; $t = 0$

$$\ln[T_0 - T_\infty] = 0 + C_1$$

$$C_1 = \ln[T_0 - T_\infty]$$

Substitute C_1 in eq (1)

$$\ln[T - T_\infty] = \frac{-hA}{\rho C_p V} t + \ln[T_0 - T_\infty]$$

$$\ln[T - T_\infty] - \ln[T_0 - T_\infty] = \frac{-hA}{\rho C_p V} t$$

$$\ln\left[\frac{T - T_\infty}{T_0 - T_\infty}\right] = \frac{-hA}{\rho C_p V} t$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\left[\frac{hAt}{\rho C_p V}\right]}$$

1) A $50 \times 50\ cm^2$ aluminium slab of thickness 6mm. is at 400°c initially and it is suddenly immersed in water. So its surface temp is lowered to 50°c. determine the time required for the slab to reach 120°c Take $h = 100\ W/m^2k$.

Sol

Given data

Dimension $= 50 \times 50\ cm^2$

$= 5 \times 50 \times 10^{-4}\ m^2$

$L = 6mm = 6 \times 10^{-3}\ m$

$T_0 = 400^\circ C = 673 K$

$T_\infty = 50^\circ C = 323 K$

$T = 120^\circ C = 393 K$

Biot number $B_i = \frac{hL}{k_s}$ $\longrightarrow$ 66pg

L = significant length $L_c = \frac{\text{thickness}}{2} = \frac{6\times10^{-3}}{2}$

$L_c = 3\times10^{-3}$

$\Rightarrow$ k from data book (Pure alloy in Properties) $\rightarrow$ Pg. 1

$\rho = 2707\ kg/m^3$

$C_p = 896\ J/kg\,K$

$K = 204.2\ W/mK$.

$$B_i = \frac{hL}{k_s} = \frac{100\times3\times10^{-3}}{204.2} = 1.46\times10^{-3}$$

$B_i < 0.1$, so it is lumped parameter System.

Slab $= \frac{T-T_\infty}{T_0-T_\infty} = \exp\left[-\frac{hA_s}{C_p V.\rho}\cdot t\right]$ $\boxed{\tau = t,\ \text{time}}$

$\tau = 117.1\ sec$

1) A mildsteel sphere of 15 mm dia is planed to be cooled by an air flow at 20°C the convective heat transfer coefficient is 110 W/m²K. Calculate the following. (i) Time required to cool the sphere 700 - 150°C

(ii) Instantaneous heat transfer rate at 150°C

(iii) total energy transfer upto 150°C

Take for mildsteel $\rho = 7850$ kg/m³, $C_p = 474$ J/kg K and thermal conductivity $k = 43$ W/mK, $\alpha = 0.044$ m²/hr

Sol: Given $d = 15\,mm = 0.015\,m$.

$$T_\infty = 20°C = 293K$$

$$h = 110\ w/m^2K$$

$$T_0 = 700°C = 973K$$

$$T = 150°C = 423K$$

$$\alpha = \frac{0.044}{60\times60} = 1.22\times10^{-5}\ m^2/sec$$

$$B_i = \frac{hL}{k_s} = \frac{110\times2.5\times10^{-3}}{43} = 6.39\times10^{-3}$$

$$L = \cancel{that}\ \frac{R}{3} = \frac{7.5\times10^{-3}}{3} = 2.5\times10^{-3}\ m$$

$B_i < 0.1$, Lumped.

Lumped
$$\frac{T - T_\infty}{T_0 - T_\infty} = exp\left[-\frac{hA_s}{C_p V \rho}\tau\right]$$

$L = \frac{V}{A_s}$

$\frac{A_s}{V} = \frac{1}{L}$

then, $\frac{423-293}{973-293} = \exp\left[\frac{-110 \times \cancel{0.0706} \times t}{474 \times 2.5 \times 10^{-3} \times 7850 \cancel{t}}\right]$

$t = 139$ sec

Instantaneous heat flow

$Q = hA_s(T - T_\infty) = 110 \times \cancel{5.30}\ 10.0706 \times (423 - 293)$

$= 1086.8$ KW

$= 10.8$ W

$A = 4\pi R^2$

$= 4\pi \times (7.5 \times 10^{-3})^2$

$= \cancel{5.30 m^2}$

$= 0.0706$

$\Rightarrow$ volume $= \frac{4}{3}\pi R^3 = \frac{4}{3} \times \pi \times (0.075)^3 = 1.76 \times 10^{-3}\ m^3$

$\Rightarrow q_t = \rho c_p V \left[T - T_0\right]$

$= 7850 \times 474 \times 1.76 \times 10^{-3}(423 - 973)$

$= -36.16$ W

The negative sign shows that the heat is coming out of the sphere

Heat flow in Semi Infinite solid

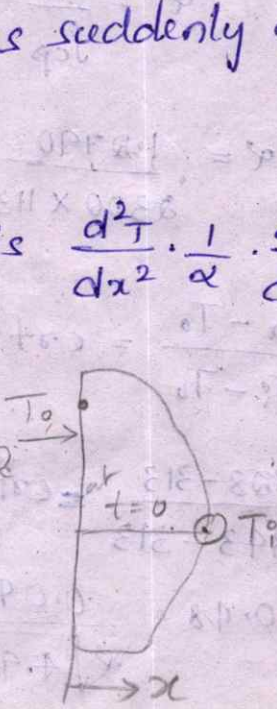
A solid which extends itself infinately in all direction of space is known as infinate solide. In an infinite solid is split into middle by a plane, each half each known as "Semi Infinite" solid."

In a semi Infinite solid at any instant of time, there is always a point, where the effect of heating or cooling at one of its boundaries. is not fell at all. At this point the temp remains unchanged.

Consider a semi-Infinite body at it ends to infinity in the positive x-direction.
The entire body is initially at uniform temp T_i including the surface at $x=0$.
The surface temp at $x=0$ is suddenly rised to T_0: –

$\Rightarrow$ The Governing equation is $\frac{d^2T}{dx^2} \cdot \frac{1}{\alpha} \cdot \frac{dT}{dt}$

$\rightarrow$ The boundary conditions are



1) $T_{(x,0)} = T_i$

2) $T_{(0,t)} = T_0$, for $t>0$

3) $T_{(\alpha,t)} = T_i$. for $t>0$

$$\frac{T_x - T_0}{T_i - T_0} = \text{erf}\ \frac{x}{2\sqrt{\alpha t}}$$ (Pg. 67)

① A large concrete highway initially at a temperature of 70°c and stream water is directed on the highway so that the surface temperature is suddenly lower to 40°c. Determine the time required to reach 55°c at a depth of 4cm of surface.

$T_i = 70°C + 273 = 343\ K$

$T_0 = 40°C + 273 = 313\ K$

$T_x = 55°c + 273 = 328\ K$

$x = 4cm = 4 \times 10^{-2} = 0.04m.$

↓ time ↓ censteady state

In this problem heat transfer h is not given so that biot number equal to infinite

It is semi - infinite solid.

$$\frac{T_x - T_0}{T_i - T_0} = \text{erf}\ \frac{x}{2\sqrt{\alpha T}}$$

$$\alpha = \frac{K}{\rho c_p}$$

$K = 1.2790\ W/mK$

$\rho = 2300\ kg/m^3$

$C_p = 1130\ J/kg\,K$

↓ Pg - 21

$$\alpha = \frac{1.2790}{2300 \times 1130} = 4.921 \times 10^{-7}$$

$$\frac{T_x - T_0}{T_i - T_0} = \text{erf}\, z$$

$$\frac{328-313}{343-313} = \text{erf}\, z = \underline{0.5}\ ,\ z = 0.48 \rightarrow \text{Pg. 68}$$

$$0.48 = \frac{0.04}{2\sqrt{4.921 \times 10^{-7} \times t}} \Rightarrow t = 3528.6\ sec$$

$t = 30 sec$ $T_x = ?$

$$\frac{T_x - T_0}{T_i - T_0} = erf \frac{x}{2\sqrt{\alpha t}}$$

$z = 1.64$, $erf = 0.97$

$$erf\, z = \frac{T_x - T_0}{T_i - T_0} \Rightarrow 0.97 = \frac{T_x - 313}{343 - 313}$$

$$T_x = 342.1\,K.$$

Infinite solid:

A solid which extends it's self infinately in all direction of space is a infinite solid.

The heat transfer coefficient b/w the surface of the plate and the fluid on both sides is assumed to be constant. The center of the plate is selected as the origin.

⇒ The governing differencial eqn

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\frac{dT}{dx}$$

① At $t = 0$, $T_0 = T_i$ & $x = 0$, $\frac{dT}{dx} = 0$

$x = \pm L$, $kA\frac{dT}{dx} = hA(T_0 - T_\infty)$

$$\frac{T_0 - T_x}{T_i - T_\infty} = f\left[\frac{x}{L}, \frac{hL}{K}, \frac{\alpha t}{L^2}\right]$$

① A Aluminium slab of 5cm thick initially at a temperature of 400°c, It is suddenly imersed in a water at 90°C. Calculate the midplane temperature after 1 min calculate temp inside the plate at a distance 10mm from the mid plane. Take $h = 1800\ W/m^2K$

Sol Given data

thickness $L = 5cm = 5 \times 10^{-2}\ m$

$T_i = 400°C + 273 = 673\ K$

$T_\infty = 90°C + 273 = 363\ K$

$t = 1min = 60sec$

$x = 10mm = 10 \times 10^{-3}m$

$h = 1800\ W/m^2k$

(i) $T_x = ?$

$$\frac{T(x,t) - T_\infty}{T_0 - T_\infty}$$

curve $= \frac{x}{L} \Rightarrow L = \frac{5}{2} = \frac{0.01}{0.025} = 4$

x-axis

$= \frac{hL}{k} = 0.225$, $k = 0.22$

$= \frac{10 \times 10^{-3} \times 1800}{0.22}$

curve $= 81.8$

y-axis $= 0.98$

$\frac{hL}{k} = \frac{1800 \times 16 \times 10^{-3}}{0.98} = 18.36$

$= \frac{T_{(x/L)} - T_\infty}{T_0 - T_\infty}$

$T_0 = 421.9$

$= \frac{\left(\frac{0.01}{0.05}\right) - 363}{-363} =$

Unit - 5

Heat exchangers

A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

Classification of heat exchangers

There are several types of heat exchangers which may be classified on the basis of:

1. Nature of heat exchange process
2. Relative direction of fluid motion
3. Design & constructional features
4. physical state of fluids

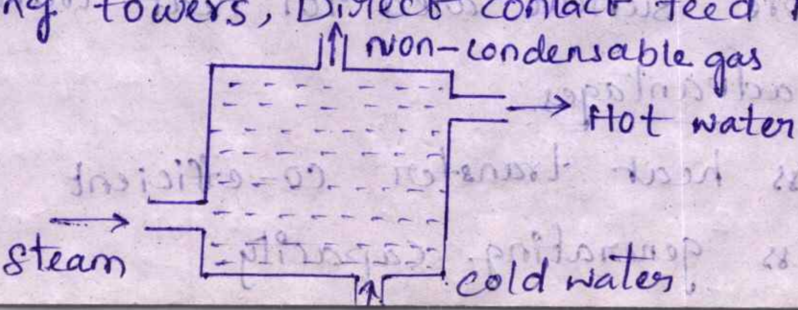
I. Nature of heat exchange Process

On the basis of the nature of heat exchange process, heat exchanger are classified as:

(a) Direct contact heat exchanger (or)
(b) Open heat exchangers:

In direct contact heat exchanger, the heat exchange takes place by direct mixing of hot & cold fluids. This heat transfer is usually accompined by mass transfer.

Eg: cooling towers, Direct contact feed heater



(b) Indirect contact heat exchangers:

In this type of heat exchangers, the transfer of heat b/w two fluids could be carried out by transmission through a walls, which separates the two fluids.

It may be classified as

(i) Regenerators

(ii) Recuperators (or) Surface heat exchangers.

(i) Regerators:

In this type of heat exchangers, hot & cold fluids flow alternately through the same space

Eg: IC engines, gas turbines.

(ii) Surface heat exchangers:

This is the most common type of heat exchanger in which the hot & cold fluid donot come into direct contact with each other but are separated by a tube wall or a surface.

Eg: Automobile radiators, Economisers etc.,

Advantages:

1) Easy construction

2) More economical

3) More surface area for heat transfer.

Dis-advantages

1) Less heat transfer co-efficient

2) Less generating capacity.

II. Relative direction of fluid motion:

This type of heat exchangers are classified as follows:

(a) Parallel flow heat exchanger
(b) Counter flow heat exchanger
(c) Cross flow heat exchanger.

(a) Parallel flow heat exchanger.

In this type, hot & cold fluids move in the same direction.

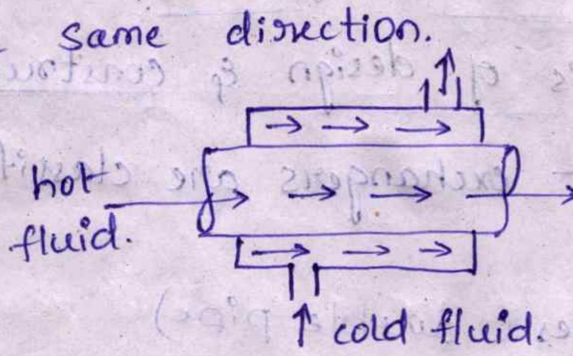


fig: Parallel flow heat exchangers.

(b) Counter flow heat exchanger:

In this types, hot & cold fluids move in parallel but opposite directions.

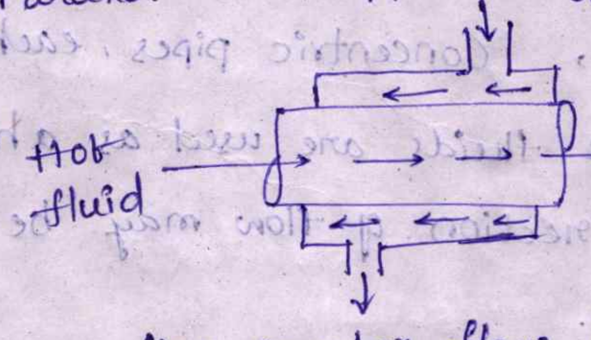


fig: Counter flow heat exchanger.

(c) Cross flow heat exchanger:

In this type, the hot & cold fluids move at right angles to each other.

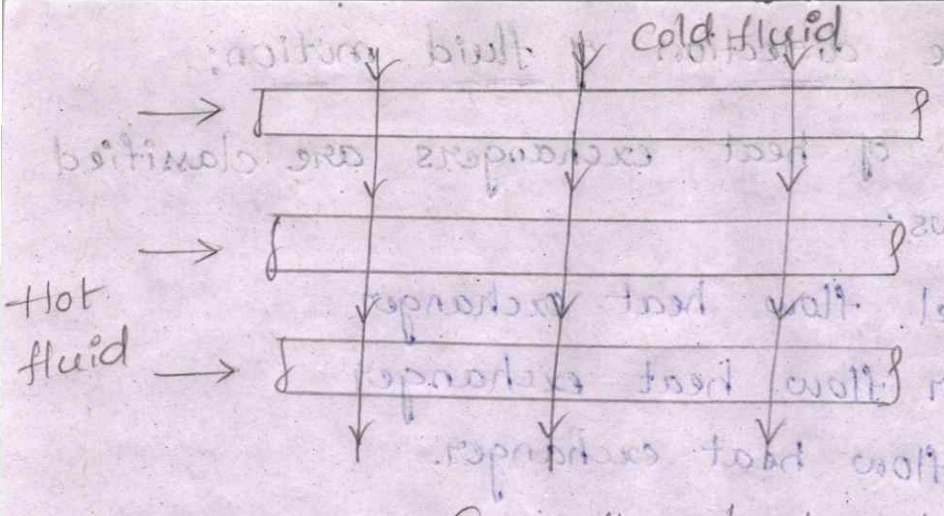


fig: Cross flow heat exchanger

III. Design & Constructional features:

On the basis of design & construction features, the Heat exchangers are classified as follows:

(a) Concentric tubes (double pipe)

(b) shell & tube

(c) multiple shell & tube passes

(d) compact heat exchanges.

(a) Concentric tubes:

In this type, Concentric pipes, each carrying one of the fluids are used as a heat exchangers. The direction of flow may be parallel or Counter

(b) Shell & Tube:

In this type of heat exchangers, one of the fluids move through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it moves over the outside surface of the tubes.

31/03/2023

In a double pipe heat exchanger, hot fluid with a specific heat of 2300 J/kg k enters at 380°c and leaves at 300°c. Cold fluid enters at 25°c and leaves at 210°c. Calculate the heat exchanger area required for Counter flow. and what would be the percentage increase in area. If the fluid flower parallel. And find mass flow rate of water.

Take overall heat transfer coefficient is 750 W/m² K and mass flow rate of hot fluid is 1 kg/sec. CPC = 4186 J/kg k, Q = 184 kW

Sol Given data

$C_{ph} = 2300$ J/kg k

$T_1 = 380°C$

$T_2 = 300°C$

$t_1 = 25°C$

$t_2 = 210°C$

Counter flow. $(\Delta T)_{lm} = \dfrac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left[\dfrac{T_1 - t_2}{T_2 - t_1}\right]}$

$$= \frac{(380 - 210) - (300 - 25)}{\ln\left[\dfrac{380 - 210}{300 - 25}\right]} = 218.3°$$

Heat exchanger $Q = UA\,(\Delta T)_{lm}$

Where $U = 750\ W/m^2 K$

$A = ?$

$$184 \times 10^3 = 750 \times A\,(218.3)$$

$$\frac{184 \times 10^3}{750} = 218.3\,A$$

$$A = \frac{184 \times 10^3}{218.3 \times 750}$$

$$\boxed{A = 1.12\ m^2}$$

(ii) Parallel flow. $(\Delta T)_{lm} = \dfrac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left[\dfrac{T_1 - t_1}{T_2 - t_2}\right]}$

$$= \frac{(380 - 25) - (300 - 210)}{\ln\left[\dfrac{380 - 25}{300 - 210}\right]}$$

$$(\Delta T)_{lm} = 193.1$$

$\Rightarrow\ 184 \times 10^3 = 750 \times A \times 193.1$

$$\boxed{A = 1.27\ m^2}$$

(iii) Percentage of Area. $= \frac{1.27 - 1.12}{1.27}$

$= 11.8 \%$

case (iii)

$Q = m_c \, Cp_c \, (t_2 - t_1) =$

$184 \times 10^3 = m_c \times 4186 \, (210 - 25)$

$m_c = 0.237 \text{ kg/s}$

~~Length of the tube~~

(2) Saturated steam at 126°C is condensing on the outer tube surface of a single pass heat exchanger. The heat exchanger heats 1050 kg/hr of water from 20°C - 95°C. The overall heat transfer coefficient is 1800 W/m²K. Calculate the following

(i) Area of heat exchanger

(ii) Length of the heat exchanger

(iii) Rate of condensation of steam.

Take $h_{fg} = 2185 \text{ kJ/kg}$, $D_1 = 0.4 \text{ m}$, $D_2 = 0.75 \text{ m}$

Sol Given data $1\,hr = 3600\,sec$

$T_1 = T_2 = 126\,°C$

$m_c = 1050\ kg/hr = \frac{1050}{3600} = 0.29\ kg/sec$

$t_1 = 20°C$

$t_2 = 95\,°C$

$T_2 > t_2$, then it is parallel flow.

$U = 1800\ W/m^2K$

$h_{fg} = 2185\ kJ/kg$ [$\because$ cp of water 4186 J/kg K]

$D_1 = 0.4\ m.$

$D_2 = 0.75\ m$

$$Q = m_h\, C_{PH}\, (T_1 - T_2) = m_c\, C_{Pc}\, (t_2 - t_1)$$

$$Q = 0.29 \times 4186 \times (95 - 20)$$

$$Q = 91045.5\ W \quad (or)\ 91 \times 10^3\ W$$

$$\Rightarrow Q = UA\, (\Delta T)_{lm}$$

$$(\Delta T)_{lm}\ \text{Parallel flow} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left[\frac{T_1 - t_1}{T_2 - t_2}\right]}$$

$$= \frac{(126 - 20) - (126 - 95)}{\ln\left[\frac{126 - 20}{126 - 95}\right]}$$

$$(\Delta T)_{lm} = 61\ °C$$

$\Rightarrow 91 \times 10^3 = 1800 \times A \times 61$

$$\boxed{A = 0.82\ m^2}$$

(ii) Length of heat exchanger.

$$\text{Area} = \pi d L = \pi \times D_1 \times L$$

$$0.82 = \pi \times 0.4 \times L$$

$$\boxed{L = 0.65\ m}$$

(iii) Rate of condensation of steam.

$$Q = m_h \times h_{fg}$$

$$m_h = \frac{Q}{h_{fg}} = \frac{91 \times 10^3}{2185 \times 10^3} = 0.0416\ kg/sec$$

(3) In a Cross flow heat exchanger both fluids ~~and~~ unmixed hot fluid with a specific heat of 2300 J/kg k. enters at 380°C and leaves at 300°C. Cold water enters at 25°C and leaves at 210°C. Calculate the required surface area of heat exchanger. Take $U = 750\ W/m^2k$ mass flow rate of hot fluid is 1 kg/sec.

Sol) Given data:

$C_h = 2300\ J/kg\ k$ $\qquad t_1 = 25°C$

$T_1 = 380°C$ $\qquad t_2 = 210°C$

$T_2 = 300°C$ $\qquad U = 750\ W/m^2k$

$m_h = 1\ kg/sec$

Counter flow $(\Delta T)_{lm} = \dfrac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left[\dfrac{T_1 - t_2}{T_2 - t_1}\right]}$

$$= \frac{(380 - 210) - (300 - 25)}{\ln\left[\dfrac{380 - 210}{300 - 25}\right]}$$

$(\Delta T)_{lm} = 218.30\ °c$

F = cross flow, Both fluids are unmixed

$x = P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{210 - 25}{380 - 25} = 0.52$

Curve = $R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{380 - 300}{210 - 25} = 0.43$

$F = 0.95$

$\boxed{Q = FUA(\Delta T)_{lm}}$

$\Rightarrow Q = m_h c_h (T_1 - T_2)$

$= 1 \times \cancel{4186}^{2300} (380 - 300)$

$Q = \cancel{334880\ W}\ 184 \times 10^3\ W$

$\Rightarrow \cancel{334880}^{184 \times 10^3} = 0.95 \times 750 \times A\ (218.30)$

$A = \cancel{2.15}\ 1.18$

(4) A parallel flow heat exchanger has hot and cold water stream running through it. The flow rate are 10 & 25 kg/min respectively. Inlet temperatures are 75°C & 25°C on hot & cold sides. The exit temperature on the hot side should not exceed 50°C. Assume $h_i = h_o = 600 \, W/m^2 k$. Calculate the area of heat exchanger using effectiveness for NTU approach

Sol Given data

$m_h = 10$ kg/min $= 0.16$ kg/sec

$m_c = 25$ kg/min $=$ $\frac{}{60}$

1 min = 60 sec

$T_1 = 75°C$

$t_1 = 25°C$

$T_2 = 50°C$

$h_i = h_o = 600 \, W/m^2 k$

$C_h = m_h c_h = 694.87 \, J/kg\,k = C_{min}$

$C_c = m_c c_c = 0.416 \times 4186 = 1741 \, W/k = C_{max}$

$$\text{effectiveness} = \varepsilon = \frac{m_h c_h}{C_{min}} \left[\frac{T_1 - T_2}{T_1 - t_1} \right]$$

$$= 0.5 = 50\%$$

Y axis = 0.5 = 50%

Cccue = $\frac{C_{min}}{C_{max}} = \frac{694}{1741} = 0.399$

x axis = ?

x-axis = 0.84 = NTU

$NTU = \frac{UA}{C_{min}}$ (or)

$\frac{1}{U} = \frac{1}{U_0} = \frac{1}{U_i}$

$= \frac{1}{}$

U = 300 W/m²K

$NTU = \frac{UA}{C_{min}} = 1.94\ m^2$.

$A = \frac{NTU \times C_{min}}{U} =$

10/04/2023

UNIT - 4

Heat transfer with Phase change.

Boiling : liquid - vapour

Condensing : Vapour - liquid

Pool boiling heat transfer phenomena.

Boiling is a convection process involving a change of phase from liquid to Vapour state.

→ According to Convection law

$$Q = hA(\Delta T)$$

ΔT = excess temperature = $T_w - T_{sat}$

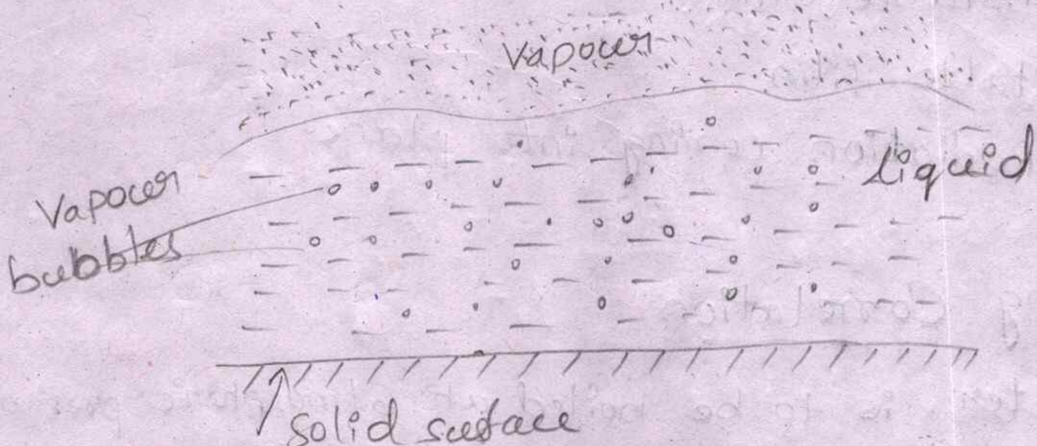


fig: Pool boiling

Q2

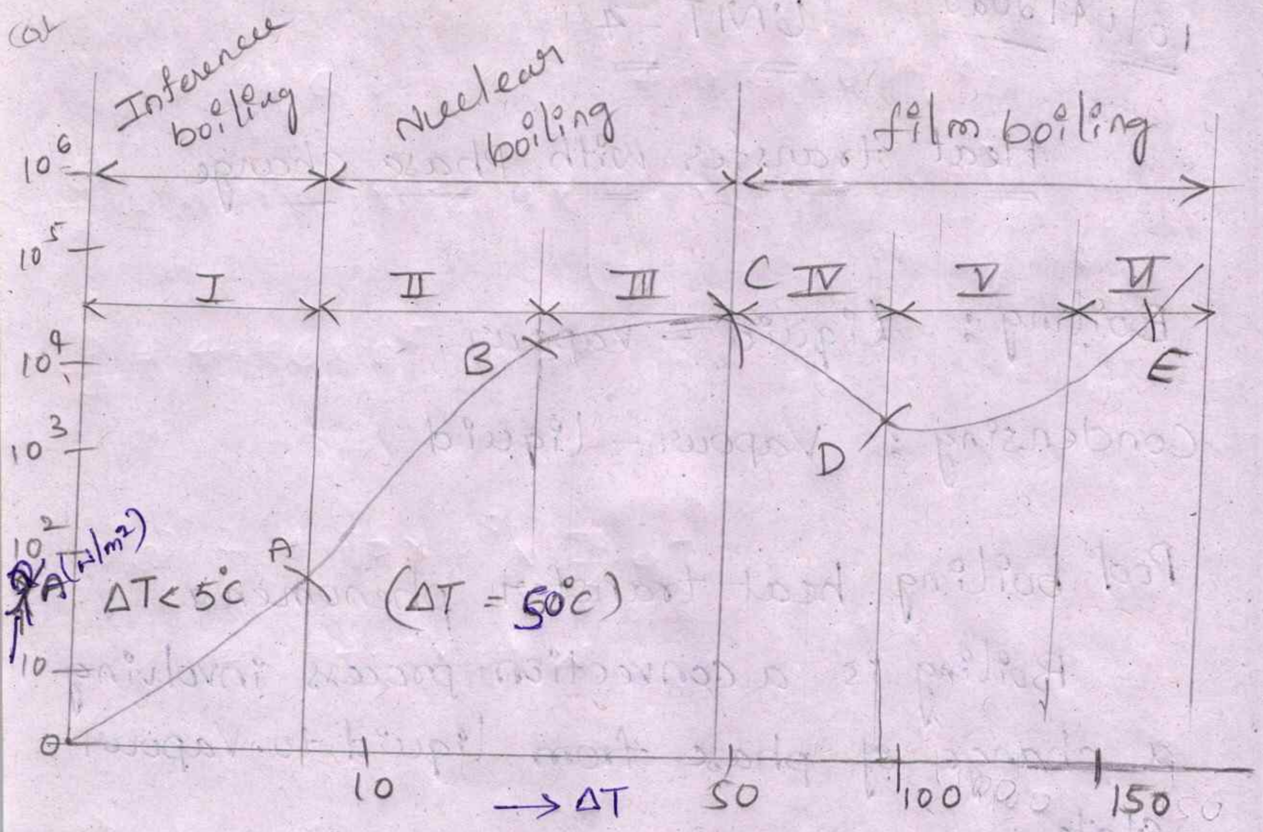


fig: Pool Boiling curve for water

1) free convection

2) bubble condensed in superheated liquid

3) bubble rise to surface.

4) unstable film

5) stable film

6) Radiation coming into plane

Boiling Correlation

① Water is to be boiled at atmospheric pressure in a polished copper pan by means of an electric heater. The diameter of the pan is 0.38m and is kept at 115°C. Calculate the following.

i) Power required to boil the water

ii) rate of evaporation.

iii) Critical heat flux.

Sol Given data

$P = 1.03$ bar $= T_{sat} = 100°C$

dia $= d = 0.38$ m.

$T_W = 115°C$

$\Delta T = T_W - T_{sat}$

$= 115 - 100$

$= 15°C$

∴ Nucleate pool boiling.

$$\frac{Q}{A} = \mu_l h_{fg} \left[g \cdot \frac{(\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[\frac{C_l \Delta T}{C_{sf} h_{fg} P_r^n} \right]^{3.0}$$

$\frac{1}{V_g}$

⇒ Property values of water at 100°C

$\rho_l = 961$ kg/m³

$V = 0.293 \times 10^{-6}$ m²/sec

$P_r = 1.740$

$\mu = \rho \times V = 961 \times 0.293 \times 10^{-6} =$

$\mu = 2.81573 \times 10^{-4}$ NS/m²

⇒ from steam tables

$h_{fg} = 2266.9$ kJ/kg

$v_g = 1.693$ m²/kg

$\rho_v = \frac{1}{v_g} = \frac{1}{1.693} = 0.597$

$\sigma = 0.0588$ N/m. ⟶

$C_{fs} = 0.013$

$n = 1$ for water , others for 0.17
$C_p = 4216\ J/kg K = 4.216\ KJ/kg K.$

$$\frac{Q}{A} = 2.81\times10^{-4} \times 2266.9 \times \left[\frac{9.81\times(961-0.597)}{0.0588}\right]^{0.5} \left[\frac{4.216\times15}{0.013\times2266.9\ 1.740}\right]^{3.1}$$

$$\frac{Q}{A} = 4.76\times10^{5}\ W/m^2$$

$$\Rightarrow Q = 4.76\times10^{5}\times\frac{\pi}{4}d^2$$
$$= 54.7\times10^{3}\ W$$

$$\Rightarrow Q = \dot{m}\times h_{fg}$$

$$m \Rightarrow \frac{Q}{h_{fg}} \text{ — } J/kgK$$

$$= 0.024\ kg/sec$$

$\Rightarrow$ critical or maximum heat flux. $1.52\times10^{6}\ W/m^2$

$$\frac{Q}{A} = 0.18\, h_{fg}\, \rho_v \left[\frac{\sigma\cdot g\cdot(\rho_l-\rho_v)}{\rho_v^2}\right]^{0.25}$$

$= 0.18\times$

$= 1.52\times10^{+6}\ W/m^2$

(2) A Nickle wire carrying electric current of 1.5mm diameter and 50cm long is submerged in a water bath which is open to atmospheric pressure. Calculate the voltage at the burn out point, if at this point the wire carries a current of 200 amperes.

Sol Given data

$d = 1.5\ mm. = 1.5 \times 10^{-3}\ m$

$l = 50\ cm = 50 \times 10^{-2}\ m.$

$I = 200\ A$

$V = ?$

$Q = V \times I$

Assume that $T_{sat} = 100°c$

from problem (1) $\frac{Q}{A}$ = Critical heat flux.

$= 1.52 \times 10^{6}\ W/m^2$

multiply & divide with 'A'

$\frac{Q}{A} = \frac{V \times I}{A}$

$1.52 \times 10^{6} = \frac{V \times 200}{\pi d l}$

$=$

$V = 17.9$ volts

$Q = V \times I$

$54.7 \times 10^{3} = V \times 200$

$V = \frac{54.7 \times 10^{2}}{200}$

$V = 273.5$

③ A heating element claded with a metal is 8mm diameter and half emmisivity is 0.92. The element is horizontally immersed in a water bath. The surface temp of the metal is 260°c under steady state boiling conditions. Calculate the power dissipation per unit length of the heater.

Sol

$d = 8\,mm = 8 \times 10^{-3}\,m$

$\varepsilon = 0.92$

$T_w = 260°C$

$Q = hA\Delta T$ $\quad A = \pi D L$

$h = ?$

$\Delta T = T_w - T_{sat}$

$= 260 - 100$

$= 160°C$

film boiling

$$h_c = 0.62\left[\frac{K_v^3 \rho_v (\rho_l - \rho_v) \cdot g\,(h_{fg} + 0.68\,C_{pv}\,\Delta T)}{\mu_v\,D\,\Delta T}\right]^{0.25}$$

$$h_r = \sigma_r\,\varepsilon\left[\frac{T_w^4 - T_{sat}^4}{T_w - T_{sat}}\right]$$

$h = h_c + 0.75 h_r$

$h = 436.02\ W/m^2 K.$

$$Q = hA\Delta T$$

$$Q = h\,\pi DL\cdot\Delta T$$

$$\frac{Q}{l} = h\,\pi D\cdot\Delta T$$

$$= 436.02 \times \pi \times 8 \times 10^{-3} \times 160$$

$$Q/l = 1753.6 \text{ W/m}$$

Condensation

Changing of phase from Vapour to Liquid

Film wise Condensation

The liquid condensate wets the solid surface spreads out and forms a film over the entire surface.

⇒ Film condensation occurs when a Vapour free from impurities.

Drop wise Condensation

The vapour condenses into small ~~ray~~ liquid droplets of various sizes. which fall down the surface in a random fraction

⇒ Dropwise Condensation heat transfer rate is 10 times higher than film wise Condensation.

Nusselt's Theory for film Condensation

The following Assumptions are made

1) The plate is maintain at the uniform temperature
2) Fluid properties are constant
3) The shear stress of the liquid vapour interface is negligible
4) The heat transfer across the condensate layer is by pure convection & the temp distribution is linear
5) The condensing vapour is entirely clean and free from gases, air and non-condensing impurities.

(1) Dry saturated steam at a pressure of 3 bar, condenses of the surface of a vertical tube of height 1m. The tube surface temp is kept at 110°C. calculate thickness of the condensate film. (ii) local heat transfer coefficient at a distance of 0.25m

Sol) $P = 3$ bar

$h = 1m$

$T_s = 110°c$ (surface)

at 3 bar from steam tables.

$T = T_v = 133.5°c$

$h_{fg} = 2163.2 \ kJ/kg. = 2163.2 \times 10^3 \ J/kg.$

$T_f = \frac{(T_v + T_s)}{2} = \frac{133.5 + 110}{2} = 121.75 \ °C$

density $\rho = 945 \ kg/m^3$

$\nu = 0.247 \times 10^{-6} \ m^2/sec$

$k = 0.6850 \ W/mk$

dynamic viscosity $\mu = \rho \times \nu$

$= 945 \times 0.247 \times 10^{-6}$

$= 2.3 \times 10^{-4} \ N\text{-}s/m^2$

thickness $\Delta x = \left[\frac{4\mu_1 k . x (T_v - T_s)}{g h_{fg} . \rho^2} \right]^{0.25}$

$x = 0.25 m.$

$$\Delta x = \left[\frac{4 \times 2.3 \times 10^{-4} \times 0.6850 \times 0.25 (133.5 - 110)}{9.81 \times 2163.2 \times 945^2 \times 10^3} \right]^{0.25}$$

$= 1.18 \times 10^{-4} \ m$

(ii) Local heat transfer coefficien (h_x)

$h_x = \frac{k}{\Delta x} = \frac{0.6850}{1.18 \times 10^{-4}} = 5805. \ W/m^2 k$

① A Vertical tube of 65 mm outside diameter and 1.5 m long is exposed to steam at atmospheric pressure. The outer surface of the tube is maintained at a temperature of 60° by circulating cold water through the tube. Calculate the following

(i) The rate of heat transfer to the coolant

(ii) The rate of condensation of steam.

(iii) Which type of flow it is ?

Sol) Given data

$d = 65\ mm = 65 \times 10^{-3}\ m.$

$L = 1.5\ m$

P = atmospheric pressure @ $T = T_v = 100°c$

$T_s = 60°c$

$$T_f = \frac{(T_v + T_s)}{2} = \frac{(100+60)}{2} = 80°C$$

properties at 100°c from steam tables.

$h_{fg} = 2256.9\ kJ/kg = 2256.9 \times 10^3\ J/kg$

at 80°c → $v = 0.364 \times 10^{-6}\ m^2/sec$

$\rho = 974\ kg/m^3$

$k = 0.6687\ W/mK.$

$\mu = \rho v = 974 \times 0.369 \times 10^{-6} = \underline{3.54 \times 10^{-4}}$ Ns/m^2

$Q = hA(T_v - T_s)$

$\Rightarrow h = 0.943 \left[\dfrac{k^3 \rho^2 g\, h_{fg}}{\mu L (T_v - T_s)} \right]^{0.25}$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc. Adams

$$h = 1.13 \left[\frac{0.6687^3 \times 974^2 \times 9.81 \times 2256.9 \times 10^3}{3.54 \times 10^{-4} \times 1.5 (100 - 60)} \right]^{0.25}$$

$\boxed{h = 4685.84 \ W/m^2K}$

$\Rightarrow A = \pi d L = \pi \times 65 \times 10^{-3} \times 1.5 = 0.3063 \ m^2$

$\Rightarrow Q = hA(T_v - T_s)$

$= 4685.8 \times 0.3063 (100 - 60)$

$\boxed{Q = 57410.42 \ W}$

(ii) The rate of Condensation of steam

$Q = \dot{m} \times h_{fg}$

$57410 = \dot{m} \times 2256.9 \times 10^3$

$\boxed{\dot{m} = 0.025 \ kg/sec.}$

(iii) Type of flow.

$$R_e = \frac{4\dot{m}}{P\mu} = \frac{4 \times 0.025}{\pi \times (0.065) \times 3.54 \times 10^{-4}}$$

$P = \pi d$

$= \pi \times 0.065$

$$\boxed{R_e = 1383.35}$$

It is less than $R_e < 1800$, Laminar flow.

(2) Steam at 0.08 bar is arranged to condensate over a 50cm square vertical plate the surface temperature is maintained at 20°C. calculate the following.

(i) Film thickness at a distance of 25cm from the top plate

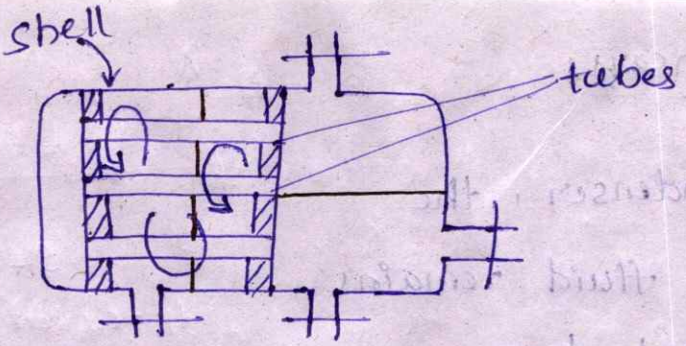
(ii) Local heat transfer coefficient at a distane of 25cm from the top of the plate.

(iii) Avg. heat transfer coefficient

(iv) Total heat transfer? (Q)

(v) Total steam condensation rate ($\dot{m}$)

(vi) What would be the heat transfer co-efficient, if the plate is inclined ° with horizontal plane.



c) Multiple shell & tube passes

In order to increase the overall heat transfer multiple shell & tube passes are used. In this, type, the 2 fluids traverse the exchanger more than one time. this type of exchanger is preferred due to its low cost of manufacture & easy to repair.

(d) Compact heat exchangers

There are many special purpose heat exchanger called compact heat exchangers. They are generally employed when convective heat transfer co-efficient associated with one of the fluids is much smaller than that associatic with the other fluid.

(iv) Physical state of fluids:

Based on the physical state of fluids inside the exchangers, heat exchangers are classified as:

(a) condensers

(b) evaporators

(a) Condensers:

In a condenser, the condensing fluid remains at constant temp throughout the exchanger while the temp. of the colder fluid gradually increased from inlet to outlet.

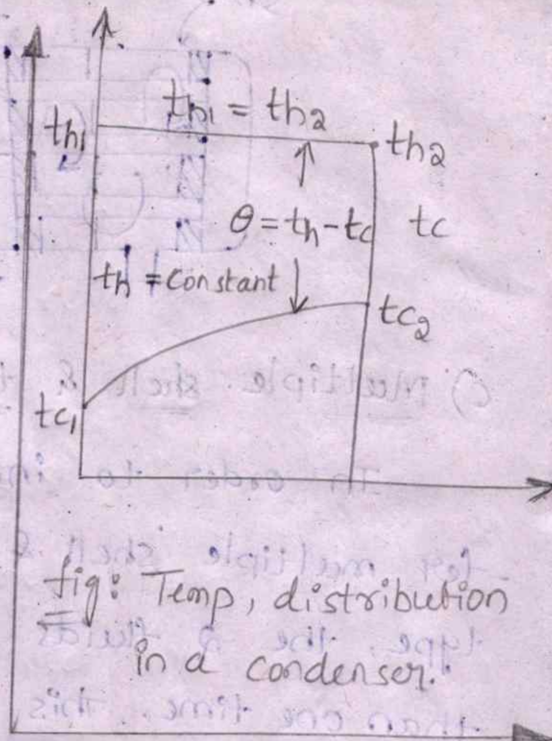
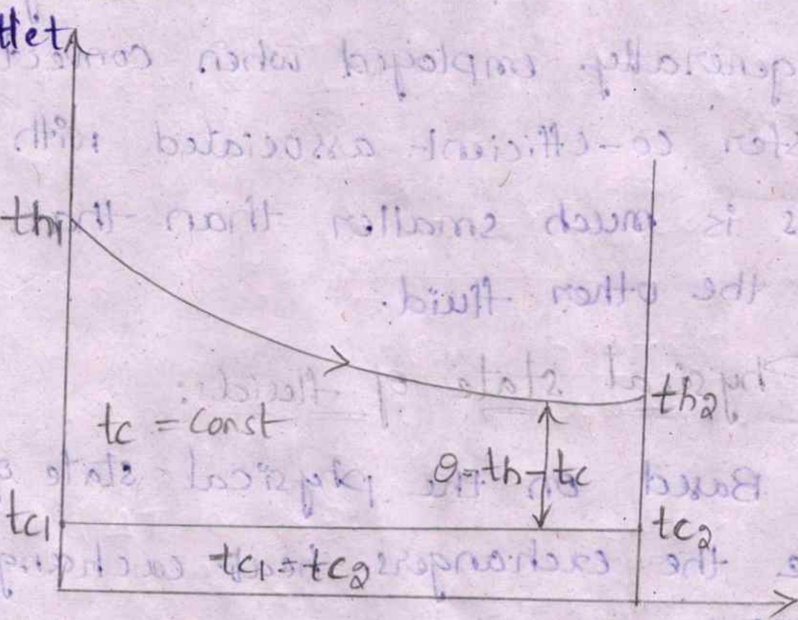


fig: Temp, distribution in a condenser.

(b) Evaporators:

In a evaporators, the cold fluid remains at constant temperature while the temperature of hot fluid gradually decreases from inlet to outlet.



⇒ Logarithmic mean temp difference (LMTD)

The temp diff b/w the hot & cold fluids in the heat exchanger varies from point to point. In addition various modes of heat transfer are involved. Therefore, based on the concept of appropriate mean temp difference also called LMTD, the total heat transfer rate in the heat exchanger is expressed as

$$Q = UA\,(\Delta T)_{lm}$$

U = overall heat transfer co-efficient, W/m^2K.

A = Area, m^2

$(\Delta T)_m$ = LMTD.

Assumptions:

1) flow is steady
2) The over all heat transfer co-efficient is constant.
3) The specific heats of both fluids are const.
4) The mass flow rate of both fluids are constant
5) Axial conduction along the tube is negligible
6) The change in kinetic & potential energies

Let m_h = mass flow rate of hot fluid

m_c = mass flow rate of cold fluid

C_{Ph} = Specific heat of hot fluid

C_{PC} = " " cold "

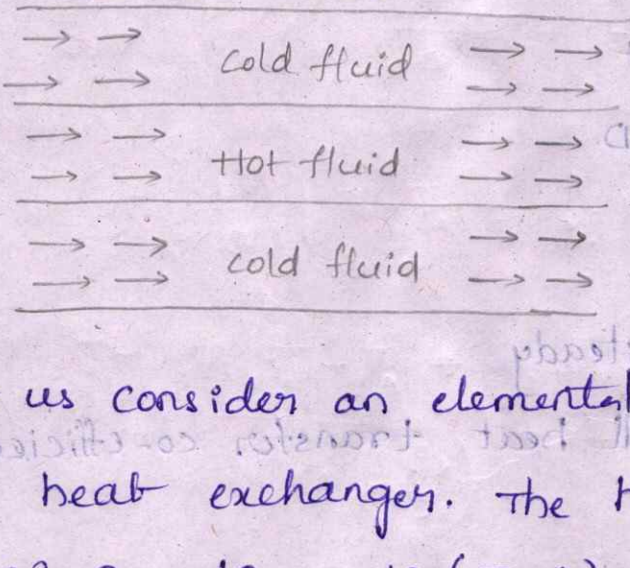
T_1 = Entry temp. of hot fluid

t_1 = " " cold "

T_2 = Exit temp of hot fluid

t_2 = " " cold "

(1) LMTD for Parallel flow.



Let us consider an elemental area dA of the heat exchanger. The heat flow rate is given $dQ = U\,dA\,(T - t) \longrightarrow (1)$

W.K.T

$dQ = -m_h C_{ph}\,dT \longrightarrow (2)$

$= + m_c\,C_{PC}\,dt$

$$dT = \frac{-dQ}{m_h C_{Ph}}$$

$$dT = \frac{-dQ}{C_h} \longrightarrow (3)$$

$\therefore\ ch = mh \times Cph$

$$dt = \frac{dQ}{mc\ CPc}$$

$$= \frac{dQ}{cc} \longrightarrow (4)$$

$\because\ Cc = mc \times CPC$

$$dT - dt = \frac{-dQ}{ch} - \frac{dQ}{cc}$$

$$d\theta = -dQ\left[\frac{1}{Ch} + \frac{1}{Cc}\right] \rightarrow (5)$$

$\because\ d\theta = dT - dt$

substituting dQ value from eq (1) in eq (5)

$$d\theta = -UdA\,(T-t)\left[\frac{1}{Ch} + \frac{1}{Cc}\right]$$

$\because\ \theta = T - t$

$$d\theta = -UdA\,\theta\left[\frac{1}{Ch} + \frac{1}{Cc}\right]$$

$$\frac{d\theta}{\theta} = -UdA\left[\frac{1}{Ch} - \frac{1}{Cc}\right]$$

Integrating

$$\int_1^2 \frac{d\theta}{\theta} = -U\left[\frac{1}{Ch} + \frac{1}{Cc}\right]\int dA$$

$$\left[\ln\theta\right]_1^2 = -U\left[\frac{1}{Ch} + \frac{1}{Cc}\right]A$$

$$\ln\theta_2 - \ln\theta_1 = -UA\left[\frac{1}{Ch} + \frac{1}{Cc}\right]$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{1}{Ch} + \frac{1}{Cc}\right] \longrightarrow (6)$$

W.K.T

$$Q = m_h\, c_{ph}(T_1 - T_2) = m_c\, c_{pc}(t_2 - t_1)$$

$$Q = C_h(T_1 - T_2) = C_c(t_2 - t_1) \longrightarrow (7)$$

$$Q = C_h(T_1 - T_2)$$

$$\frac{1}{C_h} = \frac{T_1 - T_2}{Q} \longrightarrow (8)$$

$\because C = m \times C_p$

$$Q = C_c(t_2 - t_1)$$

$$\frac{1}{C_c} = \frac{t_2 - t_1}{Q} \rightarrow (9)$$

substitute $\frac{1}{C_h}$ & $\frac{1}{C_c}$ values in eq (6)

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{T_1 - T_2}{Q} + \frac{t_2 - t_1}{Q}\right]$$

$$= \frac{-UA}{Q}\left[T_1 - T_2 + t_2 - t_1\right]$$

$$= \frac{-UA\left[T_1 - T_2 + t_2 - t_1\right]}{\ln\left(\frac{\theta_2}{\theta_1}\right)}$$

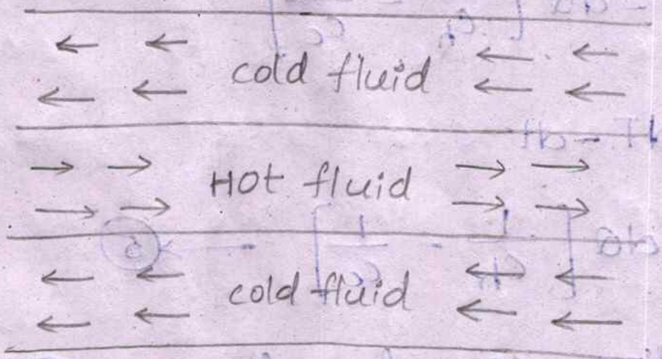
$$Q = \frac{+UA\left[T_2 - T_1 + t_1 - t_2\right]}{\ln\left(\frac{T_2 - t_2}{T_1 - t_1}\right)}$$

$$\theta = T - t \qquad = \frac{UA\left[(T_2 - t_2) - (T_1 - t_1)\right]}{\ln\left[\frac{T_2 - t_2}{T_1 - t_1}\right]}$$

$$= \frac{UA\left[(T_1 - t_1) - (T_2 - t_2)\right]}{\ln\left(\frac{T_1 - t_1}{T_2 - t_2}\right)}$$

$$Q = UA\,(\Delta T)_m.$$

(2) LMTD for counter flow.



Let us conside, on elemental area dA of the heat exchanger.

The heat flow rate is given by

$$dQ = U\,dA\,(T - t) \longrightarrow (1)$$

W.K.T

$$dQ = -m_h\,c_{ph}\,(dT) = -m_c\,c_{pc}\,(dt) \longrightarrow (2)$$

$$dQ = -m_h\,c_{ph}\,dT$$

$$dT = -\frac{dQ}{m_h\,c_{ph}}$$

$$dT = -\frac{dQ}{C_h} \longrightarrow (3) \qquad \because C_h = m_h \times c_{ph}$$

$$dQ = -m_c\, c_{pc}\, dt$$

$$dt = \frac{-dQ}{m_c\, c_{pc}}$$

$\because C_c = m_c\, c_{pc}$

$$dt = \frac{-dQ}{C_c} \quad \text{— (4)}$$

$$dT - dt = \frac{-dQ}{C_h} + \frac{dQ}{C_c}$$

$$= -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$\because d\theta = dT - dt$

$$d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \longrightarrow (5)$$

substituting dQ valve from eq (2) in eq (5)

$$d\theta = -U\,dA\,(T-t)\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$= -U\,dA\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$\because \theta = T - t$

$$\frac{d\theta}{\theta} = -U\,dA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

Integrating

$$\int_1^2 \frac{d\theta}{\theta} = -U\left[\frac{1}{C_h} - \frac{1}{C_c}\right]\int dA$$

$$[\ln \theta]_1^2 = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\ln \theta_2 - \ln \theta_1 = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \longrightarrow (6)$$

$$Q = m_h\, C_{Ph}(T_1 - T_2) = m_c\, C_{Pc}\,(t_2 - t_1)$$

$$Q = C_h\,(T_1 - T_2) = C_c\,(t_2 - t_1) \longrightarrow (7)$$

$$\because\ C = m \times C_p$$

$$Q = C_h\,(T_1 - T_2)$$

$$\frac{1}{C_h} = \frac{T_1 - T_2}{Q} \longrightarrow (8)$$

$$Q = C_c\,(t_2 - t_1) \Rightarrow \frac{1}{C_c} = \frac{t_2 - t_1}{Q} \longrightarrow (9)$$

substitute $\frac{1}{C_h}$ & $\frac{1}{C_c}$ values in eq (6)

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{(T_1 - T_2)}{Q} - \frac{(t_2 - t_1)}{Q}\right]$$

$$= \frac{-UA}{Q}\left[(T_1 - T_2) - (t_2 - t_1)\right]$$

$$= \frac{-UA}{Q}\left[(T_1 - t_2) - (T_2 - t_1)\right]$$

$$= \frac{UA}{Q}\left[(T_2 - t_1) - (T_1 - t_2)\right]$$

$$Q = \frac{UA\left[(T_2 - t_1) - (T_1 - t_2)\right]}{\ln\left(\frac{T_2 - t_1}{T_1 - t_2}\right)}$$

$$Q = UA\,(\Delta T)_m.$$

$\theta_2 = T_2 - t_1$

$\theta_1 = T_1 - t_2$

$$Q = \frac{UA\left[(T_1 - t_2) - (T_2 - t_1)\right]}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

⇒ fouling factors $\boxed{\frac{1}{U_{foul}} = R_f + \frac{1}{U_{clean}}}$

The surfaces of a heat exchangers donot remain clean after it has been in use for some time. The surface become fouled with scaling or deposits. The effect of these deposits affecting the value of overall heat transfer co-efficient (U). This effect is take care of by introducing an additional thermal registance called the fouling resistance (R_f) which is gives by as followers.

$$U_{outer} = \frac{1}{\frac{1}{h_o} + R_{fo} + \frac{r_o}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right)R_{fi} + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

$$U_{inner} = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_i}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_i}{r_o}\right)R_{fo} + \left(\frac{r_i}{r_o}\right)\frac{1}{h_o}}$$

Effectineness by using number of Trans -fer Units (NTU) :

A heat exchanger can be designed by the LMTD when inlet & outlet condition are specified. ~~But when inlet & outlet condition are specified~~. But when the problem is to determine the inlet (or) exit temp of heat exchanger, effectineness method is used

The heat exchanger effectineness is define as the ratio of actual heat transfer to the max. possible heat transfer.

$$\text{Effectineness } \epsilon = \frac{\text{Actual heat transfer}}{\text{max. possible heat transfer}}$$

$$= \frac{Q}{Q_{max}}$$

sol $P = 0.08\, bar = @ T = 41.54°C = T_v$

$A = 50 \times 50 = 2500\, cm^2 = 2500 \times (10^{-2})^2$

$T_s = 20°c$

$\Rightarrow h_{fg} = 2403.2\ kJ/kg = 2403.2 \times 10^3\ J/kg$

$\Rightarrow T_f = \dfrac{(T_v + T_s)}{2} = \dfrac{41.54 + 20}{2} = 30.77 =$

$\Rightarrow \rho = ~~997~~\ kg/m^3$

$\Rightarrow v = 0.8~~57~~ \times 10^{-6}\ m^2/~~sec~~$

$\Rightarrow k = 0.61280\ W/mK$

$\rightarrow \mu = \rho v$

$\mu = 995 \times 0.657 \times 10^{-6} = \cancel{6.53 \times 10^{-4}}\ Ns/m^2$

$\boxed{\mu = 827 \times 10^{-6}\ Ns/m^2}$

film thickness

(i) $\Delta x = \left[\dfrac{4 \mu_l\, k x\, (T_v - T_s)}{g \cdot h_{fg} \cdot \rho^2}\right]^{0.25}$

$= \left[\dfrac{4 \times 827 \times 10^{-6} \times 0.612 \times (25 \times 10^{-2})(41.54 - 20)}{9.81 \times 2403.2 \times 10^3 \times 997^2}\right]^{0.25}$

$\boxed{\Delta x = 1.46 \times 10^{-4}\, m.}$

(ii)

$\Rightarrow h_x = \dfrac{k}{\Delta x} = \dfrac{0.612}{1.46 \times 10^{-4}} = 4191.7\ W/m^2K$

(iii) Avg. heat transfer coefficient.

$$h = 0.943 \left[\frac{k^3 \rho^2 g h_{fg}}{\mu_l L (T_v - T_s)} \right]^{0.25}$$

$$= 1.13 \left[\frac{0.612^3 \times 997^2 \times 9.81 \times 2403.2 \times 10^3}{827 \times 10^{-6} \times 50 \times 10^{-2} (41.54 - 20)} \right]^{0.25}$$

$h = 5599.8 \ W/m^2K$.

(iv) Total heat transfer

$$Q = hA [T_v - T_s]$$

$= 5599.8$

$= 30139.8 \ W$

(v) steam condensation

$$\dot{Q} = \dot{m} h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} = 0.0125 \ kg/sec$$

(vi) $h_{horizontal}$ $\theta = 30°$.

$$h = 0.728 \left[\frac{k^3 \rho^2 (g \cdot \cancel{\sin} \times \cos\theta) \times h_{fg}}{\mu_l D (T_v - T_s)} \right]^{0.25}$$

$$h = 0.728 \left[\frac{0.612^3 \times 9.81 (\cos 30°) \times 2403.2 \times 10^3}{827 \times 10^{-6} \times 50 \times 10^{-2} (41.54 - 20)} \right] = 3480 \ W/m^2K$$

21/04/23

Radiation

The heat transfer from one body to another body without any transmitting medium is known as Radiation

* It is an electromagnetic wave phenomena
* these waves are classified interms of Wave length and are propogated at the speed of light i.e., 3×10^8 m/sec

Emmision Properties:

*) The rate of emission of radiation by a body depends upon the following factors.

a) the wavelength of frequency of radiation
b) the temp of the surface
c) the Nature of the surface

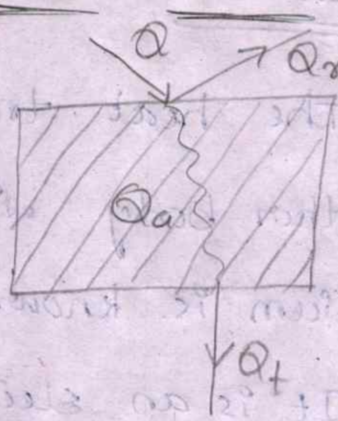
Emissive Power $[E_P]$ = W/m²

The emissive power is defined as the total amount of radiation emitted by a body per unit time & unit area.

Mono chromatic emissive power $[E_{b\lambda}]$

The energy emitted by the surface at a given length per unit time per unit area in all directions.

Absorption, Reflection & Transmission:



$$Q = Q_a + Q_r + Q_T$$

$$\frac{Q}{Q} = Q_a + Q_r + Q_\tau$$

$$\boxed{1 = \alpha + \rho + \tau}$$

absorptivity $\alpha = \dfrac{\text{Radiation absorbed}}{\text{Incident radiation}}$

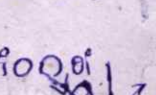
Reflectivity $\rho = \dfrac{\text{Radiation reflected}}{\text{Incident radiation}}$

Transmissivity $\tau = \dfrac{\text{Radiation Transmitted}}{\text{Incident Radiation}}$

⇒ There are two types of reflection phenomena they are:

a) Specular Reflection $\theta_i = \theta_r$ smooth surface

b) Diffused Reflection



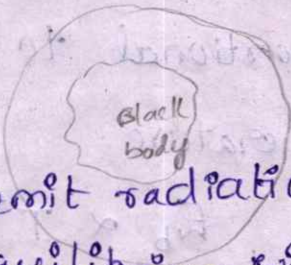
Concept of Black Body

Black body is an ideal surface having the following properties.

⇒ A black body absorbs all incident radiation, regardless of wavelength & direction.

⇒ For a prescribed temperature & Wavelength, NO surface can emitt more energy than black body. easy electronics website

An interesting point note here
↳ black body continuous to emit radiation even when it is in thermal equilibrium with its surroundings.



Plancks Distribution law

The relationship between the monochromatic emissive power of a black body & wavelength of a radiation at a particular temperature.

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$C_1 = 0.374 \times 10^{-15}$ W m^2

$C_2 = 14.4 \times 10^{-3}$ m/sec

Wien's Displacement law

The relationship between the temperature and wavelength corresponding to the maximum emissive power at a black body

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ mk} \quad \left[\because \underset{\text{micro}}{\mu} = 10^{-6} \text{ m}\right]$$

$$\lambda_{max} T = 2898 \ \mu\text{mk}$$

Stefen Boltzman law:

The emissive power of a black body is proportional to the forth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

Maximum Emissive power

A combination of plancks law and Wien's displacement law yield's the condition for the max. monochromatic emission power for a black body.

$$\left(E_{b\lambda}\right)_{max.} = C_4 T^5$$

$$C_4 = 1.307 \times 10^{-5}$$

where C_4 = (Radiation constant)

Emissivity: ratio of non black body to the black body of emissive power.

$$\epsilon = \frac{E}{E_b}$$

Grey body:

Kirchoff's law of Radiation:

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2}$$ α = absorbvity.

$$E_1 = \alpha_1$$

Intensity of radiation

$$I_n = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

① A black body at 3000k emits radiation calculate the following

(i) Monochromatic emissive power at 1 μm

(ii) wavelength at which emmissive is max.

(iii) Max. emmissive power

(iv) Total emmisive power.

(v) Calculate the total emissive of the furnace if it is assumed as a real surface having emissivity equal to 0.85

Sol Surface temp $T = 3000k$

wavelength $\lambda = 1\ \mu m$.

$\lambda = 1 \times 10^{-6} m$.

Γ

(i) $$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$= \frac{0.374 \times 10^{-15} \times (1 \times 10^{-6})^{-5}}{e^{\left[\frac{0.014387752}{1 \times 10^{-6} \times 3000}\right]} - 1}$$

$$E_{b\lambda} = 3.11 \times 10^{12} \ W/m^2$$

(ii) $$\lambda_{max} T = 2898 \times 10^{-6} \ mK$$

$$\lambda_{max} = \frac{2898 \times 10^{-6}}{3000}$$

$$= 9.6 \times 10^{-7}$$

(iii) $$\cancel{E = \varepsilon \sigma T^4}$$

$$(E_{b\lambda})_{max} = C_4 \times T^5$$

$$= 1.307 \times 10^{-5} \times T^5$$

$$= 1.307 \times 10^{-5} \times 3000^5$$

$$(E_{b\lambda})_{max} = 3.17 \times 10^{12}$$

(iv) $E_b = \sigma T^4$

$$= 5.67 \times 10^{-8} \times 3000^4$$

$$\therefore E_b = 4.59 \times 10^6 \ W/m^2$$

(v) Total emmissive power

$$(E_b)_{rad} = \varepsilon \sigma T^4$$

$(\because \epsilon = 0.85)$

$$(E_b)_{rad} = 0.85 \times 4.59 \times 10^6$$

$$(E_b)_{rad} = 3.9 \times 10^6 \ W/m^2$$

② A Black body of 1200 cm² emitts radiation at 1000K. calculate the following

(i) Total rate of energy emission

(ii) intensity of normal radiation

(iii) wave length of max. Monochromatic emmissive power.

(iv) Intensity of radiation along a direction at 60° to the normal.

Sol Given data

$$A = 1200 \text{ cm}^2$$

$$= (1200 \times 10^{-2})^2$$

$$T = 1000 K.$$

(i) Total rate of enegy emission.

$$E_b = \sigma T^4 \times A$$

$$= 5.67 \times 10^{-8} \times 1000^4 \times \left[1200 \times (10^{-2})^2\right]$$

$$= 6804 \text{ W}$$

(ii) Intensity of normal radiation

$$I_n = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

$$= \frac{5.67 \times 10^{-8} \times 1000^4}{\pi}$$

$$= 18048 \text{ W/m}^2$$

(iii) wavelength of Max. monochromatic emission power

$$\lambda_{max} T = 2898 \ \mu mK$$

$$\lambda_{max} = \frac{2898}{1000}$$

$$\lambda_{max} = 2.89 \ \mu$$

(iv)

$$I_n = I_\theta$$

$$= 18.048 \text{ W/m}^2$$

(3) Assuming sun to be black body emitting radiation at 6000K at a mean distance of 12×10^{10} m for the earth. the diameter of sun is 1.5×10^{9} m. and that of the earth is 13.2×10^{6} m. calculate the following.

(i) Total energy emitted by the sun

(ii) The emmission received per m^2 just outside the earth atmosphere.

(iii) The total energy received by the earth if no radiation is blocked by the earth atmosphere

(iv) The energy received by a 2x2 m solar collector whose normal is inclined at 45^0 to the sun. The energy loss through the atmosphere is 50% and the diffused radiation is 20% of direct radiation.

Sol) Given data

$T = 6000K$

$R = 12 \times 10^{10}$ m

$D_{sun} = 1.5 \times 10^{9}$ m

$D_{earth} = 13.2 \times 10^{6}$ m.

(i) $E_b = \sigma T^4 . A$

$= \sigma T^4 \; 4\pi (R_{sun}^2)$

$= 5.67 \times 10^{-8} \times 6000^4 \times 4\pi \left(\frac{1.5 \times 10^9}{2}\right)^2$

$= 5.19 \times 10^{26}$ W

ii) $E_{b(earth\ atm)} = \frac{E_b}{A}$

$= \frac{E_b}{4\pi R^2_{distance}}$

$= \frac{5.19 \times 10^{26}}{4\pi \times (12 \times 10^{10})^2}$

$= 2868.10 \ W/m^2$

iii) $E_{b\,(earth\ atm)} \times A$

$= 2868.10 \times \frac{\pi}{4}(d^2)$

$= 2868.10 \times \frac{\pi}{4}(13.2 \times 10^6)^2$

$= 3.92 \times 10^{17} \ W$

iv) Energy loss through atmosphere is 50%

$E_1 = \frac{50}{100} \times 2868$

$E_1 = 1434 \ W/m^2$

The diffused radiation is 20% of direct radiation

$E_2 = 1434 \times \frac{20}{100}$

$= 286.8 \ W/m^2$

Total radiation reaching the collector

$E = E_1 + E_2$

$= 286.8 + 1434$

$E = 1720.8 \ W/m^2$

Energy received by the solar panel

$$E \times A \times \cos\theta$$

$$= 1720 \times 2 \times 2 \times \cos(45^\circ)$$

$$= 48648.9 \text{ W}$$

(4) 800 W/m² of radiant energy is Incident upon a surface of out of which 300 W/m² is absorbed, 100 W/m² is reflected & the remainder is Transmitted. Calculate the following. (i) Absorptivity (ii) Reflectivity (iii) Transmitivity.

sol

$$Q = Q_\alpha + Q_r + Q_\tau$$

$$800 = 300 + 100 + Q_\tau$$

$$Q_\tau = 400 \text{ W/m}^2$$

(i) Absorptivity $= \frac{Q_\alpha}{Q} = \frac{300}{800} = 0.375$

(ii) Reflectivity $= \frac{Q_r}{Q} = \frac{100}{800} = 0.125$

(iii) Transmitivity $= \frac{Q_\tau}{Q} = \frac{400}{800} = 0.5$

$$\boxed{\alpha + r + \tau = 1}$$

(5) Radiation exchange b/w Surfaces

Radiant energy exchange between surfaces depends not only on the emission, absorption and reflection. characteristics of the surfaces but also on their geometrical arrangements.

* This Heat exchange will be effect further due to the presence partially emitting & absorping medium between the surfaces.

* To account this radiation exchange following assumptions are made

⇒ All the surfaces are considered to be either black or Grey.

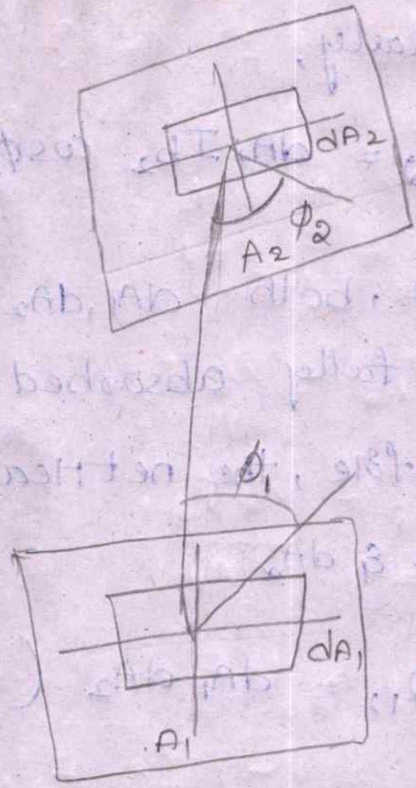
⇒ Radiation and reflection process are assumed to diffusion.

⇒ The absorpvity of surface is taken equals to it emissivity & independent of temperature of the source of the Incident radiation.

Radiation Exchange Between two black Surfaces separated by a Non-absorbing medium

(402) pg.

Let us consider two black bodies separated by an non-absorbing medium.



⇒ The problem is to determine the Net radiation exchange between them.

⇒ Consider Area elements dA_1 & dA_2 are the two surfaces the distance b/w them is 'r'

⇒ & The angle normals two area elements make with the line joining them are ϕ_1 & ϕ_2

⇒ The rate of radiative energy dQ, Living dA_1 that strikes dA_2 is given by

$$dQ_1 = dA_1\, I_{b_1} \cos\phi_1\, d\omega_{12}$$

Where solid angle

$$\cancel{d\omega_{12}} = \frac{\cancel{dA_1 I_{b_1} \cos\phi_1}}{\cancel{d\theta}}$$

$$d\omega_{12} = \frac{dA_2 \cos\phi_2}{r^2}$$

$$dQ_1 = dA_1\, I_{b_1} \cos\phi_1\, \frac{dA_2 \cos\phi_2}{r^2}$$

Similarly,

$$dQ_2 = dA_2 I_{b_2} \cos\phi_2 \cdot \frac{dA_1 \cos\phi_1}{r^2}$$

Since, both dA_1, dA_2 black surface, dQ_1 & dQ are fully absorbed by dA_1 & dA_2 respectively

Therefore, the net Heat transfer energy between dA_1 & dA_2

$$dQ_{12} = dA_1\, dA_2 \left(I_{b_1} - I_{b_2}\right) \frac{\cos\phi_2 \cos\phi_1}{r^2}$$

$\because I_b = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$

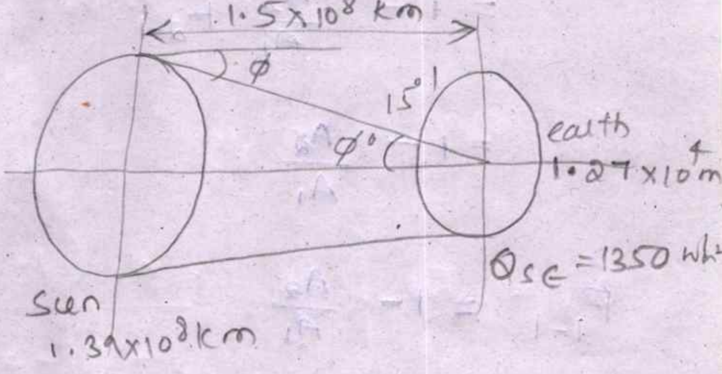
$$dQ_{12} = \left(\frac{E_{b_1}}{\pi} - \frac{E_{b_2}}{\pi}\right)(dA_1\, dA_2)\left(\frac{\cos\phi_2 \cos\phi_1}{r^2}\right)$$

The net energy exchange between A_1 & A_2 is obtained by integrating

$$\boxed{Q_{12} = \iint_{A_1 A_2} \frac{\sigma}{\pi r^2}\left(T_1^4 - T_2^4\right) dA_1\, dA_2 \cos\phi_1 \cos\phi_2}$$

Shape Factor :

① 800 W/m² radiant energy is Incident upon a surface, out of its which 300 W/m² is observed, ① The sun is a spherical mass of extremely hot gas continuously generating heat by thermo neuclear fusion reaction. This energy is radiated from the sun in all directions and a small fraction of it reaches the earth. the orientation of sun and earth is known in fig. on a clear day, the solar radiation on the earth has been found to be 1350 W/m². Assuming sun to be a black body. estimate its temperature.



Sol

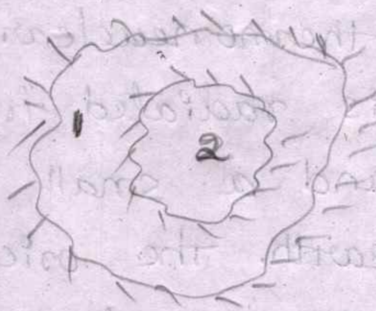
$$E_b = \sigma T_s^4 = \frac{d\,Q_{SE}}{d\,Q_E}$$

$$= \frac{\pi r^2}{\frac{\pi}{4} r^2}$$

$$= \frac{\pi (1.5 \times 10^{11})^2 \times 1350}{\frac{\pi}{4} (1.39 \times 10^9)^2}$$

$$T_s = 5400^\circ C$$

② Calculate the shape factor for the confi-guration shows in the fig.



$$F_{1+1} + F_{1-2} = 1$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$F_{1-2} = \frac{A_2 F_{2-1}}{A_1}$$

$$F_{1-1} = 1 - F_{1-2}$$

$$= 1 - \frac{A_2}{A_1} F_{2-1}$$

$$= 1 - \frac{A_2}{A_1}$$

$$F_{1-1} = 1 - \frac{A_2}{A_1}$$

$$F_{2-1} = 1$$

$$F_{1-1} = 1 - \frac{A_2}{A_1}$$

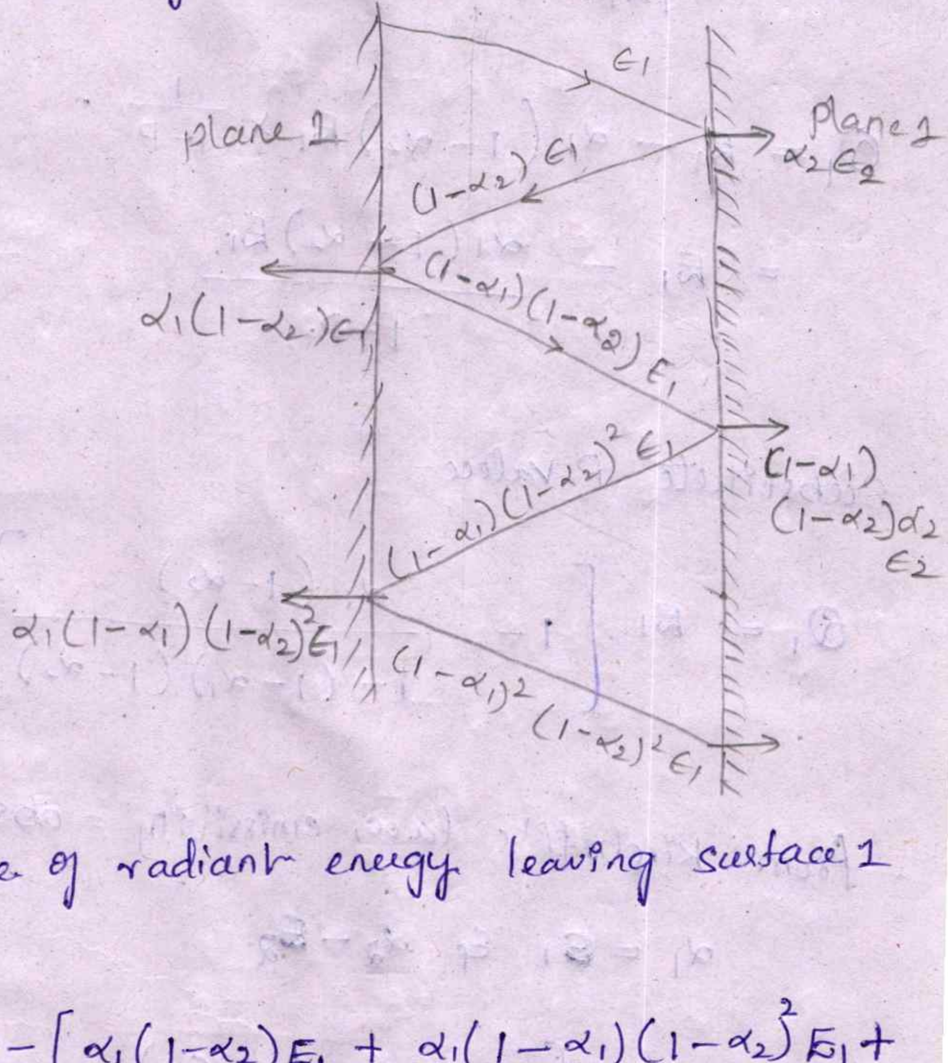
$$F_{1-2} = \frac{A_2 F_{2-1}}{A_1}$$

$$F_{2-1} = 1$$

③

Radiation ~~sheild~~ shield (442 Pg)

consider two very large parallel gray surfaces of areas A_1 and A_2, at a small distance apart, and exchanging radiation as shown in Fig.



the rate of radiant energy leaving surface 1 is

$$Q_1 = E_1 - [\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \cdots]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \cdots]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1[1 + P + P^2 + \cdots]$$

where $P = (1-\alpha_1)(1-\alpha_2)$

Since α_1 & α_2 are less than 1, as $P<1$

$1 + P + P^2 + \cdots\cdots$ when extended to infinity

$$= \frac{1}{1-P}$$

$$Q_1 = E_1 - \alpha_1(1-\alpha_2)E_1 \times \frac{1}{1-P}$$

$$= E_1 - \frac{\alpha_1(1-\alpha_2)E_1}{1-P}$$

substitute P value

$$Q_1 = E_1\left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

from Kirchoff's law, emissivity = absorptivity

$\alpha_1 = \varepsilon_1$ & $\alpha_2 - \varepsilon_2$

$$\Rightarrow Q_1 = E_1\left[1 - \frac{\varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$= E_1\left[\frac{1-(1-\varepsilon_1)(1-\varepsilon_2) - \varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$= E_1 \left[\frac{1 - (1 - \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2] - \epsilon_1 (1 - \epsilon_2)}{1 - [1 - \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2)} \right]$$

$$= E_1 \left[\frac{1 - 1 + \epsilon_2 + \epsilon_1 - \epsilon_1 \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2}{1 - 1 + \epsilon_2 + \epsilon_1 - \epsilon_1 \epsilon_2} \right]$$

$$= \frac{E_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$\boxed{Q_1 = \frac{E_1 \epsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}}$$

the net radiative heat exchange from surface 1-2 is given by

$$Q_{12} = Q_1 - Q_2$$

$$= \frac{E_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} - \frac{E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$Q_{12} = \frac{E_1 \epsilon_2 - E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$E_b = \sigma T^4$$

$$E_1 = \sigma T_1^4$$

$$\boxed{\begin{aligned} E_1 &= \varepsilon_1 \sigma T_1^4 \\ E_2 &= \varepsilon_2 \sigma T_2^4 \end{aligned}}$$

Substitute E_1 & E_2 values in eq

$$Q_{12} = \frac{\varepsilon_1 \sigma T_1^4 \varepsilon_2 - \varepsilon_2 \sigma T_2^4 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2 \sigma \left[T_1^4 - T_2^4\right]}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \times \sigma \left[T_1^4 - T_2^4\right]$$

$$Q_{12} = \bar{\varepsilon} \sigma \left[T_1^4 - T_2^4\right]$$

where $\Rightarrow \bar{\varepsilon} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$

Divide by $\varepsilon_1 \varepsilon_2$,

$$\Rightarrow \bar{\varepsilon} = \frac{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 \varepsilon_2}}{\frac{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}{\varepsilon_1 \varepsilon_2}}$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1} - 1}$$

$$\boxed{\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}}$$

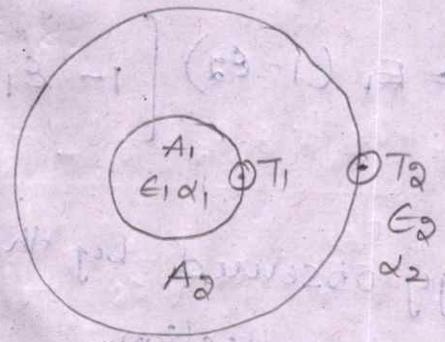
Heat exchange between two parallel surfa[ces] is given by (considering area)

$$Q_{12} = \bar{\epsilon}\,\sigma A\left[T_1^4 - T_2^4\right]$$

$$\bar{e} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Heat exchange between two large Concentric cylinders or spheres (427)

Consider two large Concentric cylinders of area A_1 & A_2



$$F_{12} A_1 = F_{21} A_2$$

$$F_{12} = 1$$

$$\boxed{F_{21} = \frac{A_1}{A_2}}$$

Considering the energy emitted by the inner cylinder.

① inner cylinder emits the energy $= E_1$

② outer cylinder absorbs energy $= \alpha_2 E_1$

$= \epsilon_2 E_1$

③ Outer cylinder reflects energy $= E_1(1-\epsilon_2)$

④ Inner cylinder absorbs energy $= E_1(1-\epsilon_2)F_{21}\alpha_1$

$$= E_1(1-\epsilon_2)\frac{A_1}{A_2}\epsilon_1$$

$$\left[\because F_{21} = \frac{A_1}{A_2},\ \alpha_1 = \epsilon_1\right]$$

⑤ Inner cylinder reflects energy

$$= E_1(1-\epsilon_2) - E_1(1-\epsilon_2)\epsilon_1\frac{A_1}{A_2}$$

$$= E_1(1-\epsilon_2)\left[1-\epsilon_1\frac{A_1}{A_2}\right]$$

⑥ Energy observed by the inner cylinder on the second reflection.

$$= E_1(1-\epsilon_2)^2\epsilon_1\frac{A_1}{A_2}\left[1-\frac{A_2}{A_1}\epsilon_1\right] + \text{———}$$

$$Q_1 = E_1 - E_1(1-\epsilon_2)\epsilon_1\frac{A_1}{A_2} - E_2(1-\epsilon_2)^2\epsilon_1\frac{A_1}{A_2}\left[1-\frac{A_1}{A_2}\epsilon_1\right] + \cdots$$

$$= E_1\left[1-\frac{A_1}{A_2}\epsilon_1(1-\epsilon_2)\left\{1+(1-\epsilon_2)\left(1-\frac{A_1}{A_2}\epsilon_1\right) + (1-\epsilon_2)^2\times\left(1-\frac{A_1}{A_2}\epsilon_1\right)^2 + \cdots\right\}\right]$$

$$E_{b1}\left[1-\frac{\frac{A_1}{A_2}\varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_2)\left[1-\frac{A_1}{A_2}\varepsilon_1\right]}\right]$$

$$\boxed{Q_1 = \frac{E_{b1}\,\varepsilon_2}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}}$$

Similarly

$$Q_2 = \frac{E_{b2}\,\varepsilon_1\,\frac{A_1}{A_2}}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}$$

The net radiation heat transfer between the inner & outer concentric cylinders is given by

$$Q_{12} = Q_1 - Q_2$$

$$Q_{12} = \frac{E_{b1}\,\varepsilon_2}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2} - \frac{E_{b2}\,\varepsilon_1\,\frac{A_1}{A_2}}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}$$

Considering area A_1 & A_2

$$\Rightarrow Q_{12} = \frac{A_1 E_{b1}\,\varepsilon_2}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\left(\frac{A_1}{A_2}\varepsilon_1\varepsilon_2\right)} - \frac{A_2 E_{b2}\,\varepsilon_1\,\frac{A_1}{A_2}}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}$$

$$\Rightarrow Q_{12} = \frac{A_1 E_{b1}\,\varepsilon_2 - (A_1 E_{b2}\,\varepsilon_1)}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2} \longrightarrow (a)$$

from stefan Boltzman law, we know that,

$$E_b = \varepsilon \sigma T^4$$

$$E_1 = \varepsilon_1 \sigma T_1^4$$

$$E_2 = \varepsilon_2 \sigma T_2^4$$

Substituting E_1 & E_2 in eq

$$Q_{12} = \frac{A \varepsilon_1 \sigma T^4 \varepsilon_2 - A_1 \varepsilon_2 \sigma T_2^4 \varepsilon_1}{\frac{A_1}{A_2}\varepsilon_1 + \varepsilon_2 - \frac{A_1}{A_2}\varepsilon_1 \varepsilon_2}$$

$$= \frac{A_1 \sigma \varepsilon_1 \varepsilon_2 [T_1^4 - T_2^4]}{\left[\frac{A_1}{A_2}\varepsilon_1 \varepsilon_2 \left(\frac{1}{\varepsilon_2} - 1\right)\right] + \varepsilon_2}$$

Dividing by $\varepsilon_1 \varepsilon_2$

$$Q_{12} = \frac{A_1 \sigma [T_1^4 - T_2^4]}{\frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{1}{\varepsilon_1}}$$

$$Q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

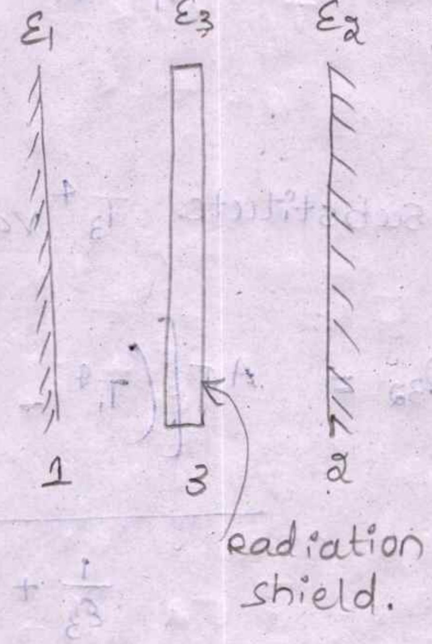
$$Q_{12} = \bar{\varepsilon} A_1 \sigma (T_1^4 - T_2^4)$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

for cylinder, Area $A = 2\pi r L$

for sphere $A = 4\pi r^2$

Radiation shield (442)



The net heat Exchange between parallel plates without radiation shield

$$Q_{12} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Heat exchange between 1-3

$$Q_{13} = \frac{A\sigma\left(T_1^4 - T_3^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \longrightarrow (a)$$

Heat exchange between 3-2

$$Q_{32} = \frac{A\sigma\left(T_3^4 - T_2^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \longrightarrow (b)$$

from (a)

$$T_1^4 - T_3^4 = \frac{Q_{13}\left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right]}{A\sigma}$$

$$\Rightarrow T_3^4 = T_1^4 - \frac{Q_{13}\left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right]}{A\sigma}$$

substitute T_3^4 value in (b)

$$Q_{32} = \frac{A\sigma\left[\left(T_1^4 - \frac{Q_{13}\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}{A\sigma}\right] - T_2^4\right.}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow Q_{32}\left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right) = A\sigma T_1^4 - Q_{13}\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) - A\sigma T_2^4$$

under equilibrium condition

$$Q_{13} = Q_{32}$$

$$Q_{13}\left[\left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) = A\sigma\left(T_1^4 - T_2^4\right)\right.$$

$$Q_{13} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)} \longrightarrow (c)$$

Dividing the equation (c) in eq (a)

$$\frac{Q_{13}}{Q_{12}} = \frac{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)}{\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

$$\frac{Q_{13}}{Q_{12}} = \frac{1}{2}$$

$Q_{13} = \frac{1}{2} Q_{12}$ (or) $Q_{32} = \frac{1}{2} Q_{12}$

Thus by inserting one shield between two parallel surfaces the direct radiation heat transfer between them is halved.

① Calculate net radiant interchange for square meter. for two large planes at a temperature of 900K and 400k respectively. Assume that the emissivity of hot plane is 0.9 and that of cold plane is 0.7.

Sol

$T_1 = 900K$ $\qquad \varepsilon_1 = 0.9$

$T_2 = 400K$ $\qquad \varepsilon_2 = 0.7$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.9} + \frac{1}{0.7} - 1} = 0.649$$

$$\frac{Q_{12}}{A} = \bar{\varepsilon}\,\sigma\left[T_1^4 - T_2^4\right]$$

$$\frac{Q_{12}}{A} = 0.649 \times 5.67 \times 10^{-8}\left[900^4 - 400^4\right]$$

$$= 23.20 \text{ kW/m}^2$$

② Two large parallel plates are maintained at a temperature of 900k & 500k respectively. Each plate has an area of 6 m². Compare the net heat exchange between the plates for the following cases. (i) both plates are black ii) plates of an emissivity of 0.5.

Sol) $T_1 = 900K$

$T_2 = 500K$

$A = 6m^2$

(i) both plates are black

$\varepsilon = 1$

$$Q_{12} = \bar{\varepsilon}\sigma A(T_1^4 - T_2^4)$$

$$= 1 \times 5.67 \times 10^{-8} \times 6 \times (900^4 - 500^4)$$

$$= 201942.7 \text{ W}$$

(ii) plates of an emissivity

$\varepsilon_1 = \varepsilon_2 = 0.5$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.5} - 1} = 0.33$$

$$Q_{12} = \bar{\varepsilon}\sigma A \times (T_1^4 - T_2^4)$$

$$= 0.33 \times 5.67 \times 10^{-8} \times 6 \times (900^4 - 500^4)$$

$$= 66641.09 \text{ W}$$

3. Calculate the heat exchange by radiation between the surfaces of two long cylinders having radii 120 mm & 60 mm respectively. The axis of the cylinder are parallel to each other. The inner cylinder is maintain at

130°c and emissivity of 0.6. The outer cylinder is maintain at 30°c and emissivity of 0.5

in – out = A1

out – in = A2

Sol $r_1 = 60\times10^{-3}$ m

$r_2 = 120\times10^{-3}$ m

$T_1 = 403$ K $\qquad \mathcal{E}_1 = 0.6$

$T_2 = 303$ K $\qquad \mathcal{E}_2 = 0.5$

$L_1 = L_2 = 1$

$$\mathcal{E} = \frac{1}{\frac{1}{\mathcal{E}_1} + \frac{A_1}{A_2}\left(\frac{1}{\mathcal{E}_2} - 1\right)} = \frac{1}{\frac{1}{0.6} + \frac{0.376}{0.753}\left[\frac{1}{0.5} - 1\right]}$$

$$= 0.4616$$

$A_1 = \pi D_1 L = \pi \times 120\times10^{-3}\,m \times 1 = 0.376$

$A_2 = \pi D_2 L = \pi \times 240\times10^{-3} \times 1 = 0.753$

$$Q_{12} = \mathcal{E}\,\sigma A_1 (T_1^4 - T_2^4)$$

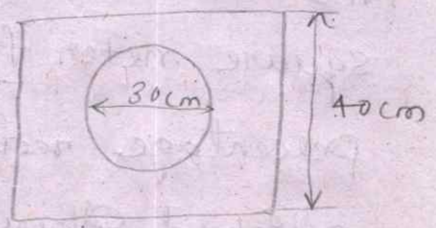
$$= 0.4616 \times 5.67\times10^{-8} \times 0.376 (403^4 - 303^4)$$

$$= 176.6 \text{ W}$$

3 (b) rate of evaporation of liquid oxygen if its rate of evaporation of latent heat is 200 kJ/kg

$$\frac{Q \rightarrow \text{J/sec}}{\text{Latent heat} \rightarrow \text{kJ/kg}} = \text{Rate of evaporation} = \frac{176}{200\times10^3} = 8.8\times10^{-4}\ \text{kg/sec}$$

(4) A pipe of outer diameter is 30cm having Emissivity 0.6 and at a temperature of 600K runs centrally in a brick duct of 40cm side square section having Emissivity 0.8 and at a temperature of 300K. Calculate the following (i) Heat exchange per meter length (ii) ~~Calculate~~ the Convective heat transfer Co-efficient when surrounding of duct is 280K.



Sol

$D_1 = 30 \times 10^{-2}\ m$

$\varepsilon_1 = 0.6$

$T_1 = 600K$

Side $= 40 \times 10^{-2}\ m$

$T_2 = 300K$

$\varepsilon_2 = 0.8$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}} = \frac{1}{\frac{1}{0.6} + \left(\frac{1}{0.8} - 1\right)\frac{0.942}{1.6}}$$

$$= 0.5513$$

$A_1 = \pi D L = \pi \times 30 \times 10^{-2} \times 1 = 0.942$

$A_2 = (0.4 \times 4) \times 1 \times 1 = 1.6\ m^2$

$$\frac{Q_{12}}{A} = \bar{\varepsilon}\sigma \times A_1 (T_1^4 - T_2^4) = 0.55 \times 5.67 \times 10^{-8} \times 0.942 \times (600^4 - 300^4)$$

$$= 3569.2\ W/m.$$

(ii) Convective heat transfer co-efficient

$$Q = hA(T_w - T_s)$$

$$3569.2 = h \times 1 \left(300 - 280\right)$$

$$h = 178.46\ W/m^2K.$$

(5) Emissivity of two large parallel plates maintain at 800°c, 300°c are 0.3 & 0.5 respectively. Find the net radiant heat exchange per square meter for these plates. Find the percentage reduction in heat transfer with a polished Aluminium radiation shield of Emissivity 0.06. ~~Find~~ is placed between them. and also Find the temperature of the shield.

Sol Given $\varepsilon_1 = 0.3$ $\varepsilon_3 = 0.06$

$\varepsilon_2 = 0.5$ $T_1 = 1073\ K$

$T_2 = 573\ K$

$$Q_{12} = \varepsilon \sigma A (T_1^4 - T_2^4)$$

$$\varepsilon = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1} = 0.23$$

$$\frac{Q_{12}}{A} = 0.23 \times 5.67 \times 10^{-8} \left(1073^4 - (300+273)\,573^4\right)$$

$$= 15.8\ kW/m^2$$

$$Q_{13} = Q_{32}$$

$$\bar{\varepsilon} A \sigma (T_1^4 - T_3^4) = \bar{\varepsilon} A \sigma (T_3^4 - T_2^4)$$

$$\frac{\cancel{A\sigma} (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\cancel{A\sigma} (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\frac{(1073^4 - T_3^4)}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = \frac{(T_3^4 - 573^4)}{\frac{1}{0.06} + \frac{1}{0.5} - 1}$$

$$T_3 = 911.5\ K$$

$$Q_{13} = \varepsilon \sigma A (T_1^4 - T_3^4)$$

$$\frac{Q_{13}}{A} = \varepsilon \sigma (T_1^4 - T_3^4)$$

$$= 0.05 \times 5.67 \times 10^{-8} (1073^4 - 911.4^4)$$

$$= \boxed{1081.86}\ W/m^2$$

$$\varepsilon_{13} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = 0.05$$

(iii) Percentage redcuction $= \dfrac{Q_{12} - Q_{13}}{Q_{12}}$

$$= \frac{15880 - 1895.7}{15880}$$

$$= 88\ \%$$

Electrical Network Analog for Thermal radiation systems

Irradiation (G) : It is defined as the total radiation incident upon the surface per unit area. ~~Radiaci~~

Radio city (J) : It is used to indicate the total radiation leaving a surface per unit time per unit area.

⇒ The radio city consists of two parts

i) Reflected by the surface $[\rho G]$

ii) Emitted by a surface $[\varepsilon E_b]$

$$J = \rho G + \varepsilon E_b$$

$$\alpha + \rho + \tau = 1 \qquad [\because \tau = 0]$$

$$\alpha + \rho = 1$$

$$\rho = 1 - \alpha \qquad [\because \alpha = \varepsilon]$$

$$J = (1-\varepsilon) G + \varepsilon E_b$$

$$J = \varepsilon E_b + (1-\varepsilon) G$$

$$J - \varepsilon E_b = (1-\varepsilon) G$$

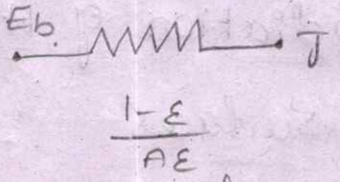
$$\boxed{G = \frac{J - \varepsilon E_b}{1-\varepsilon}}$$

$$\frac{Q_{12}}{A} = J - G$$

$$= J - \left[\frac{J - \varepsilon E_b}{1-\varepsilon} \right]$$

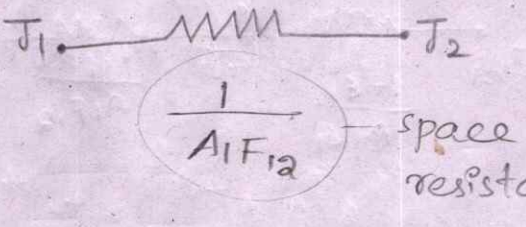
$$\frac{Q_{12}}{A} = \frac{\varepsilon (E_b - J)}{1-\varepsilon}$$

$$Q_{12} = \frac{E_b - J}{\frac{1-\varepsilon}{A\varepsilon}}$$



electrical circuit

⟹ when plate is changes direction shape Factor changes.



Overall heat transfer

$$Q_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

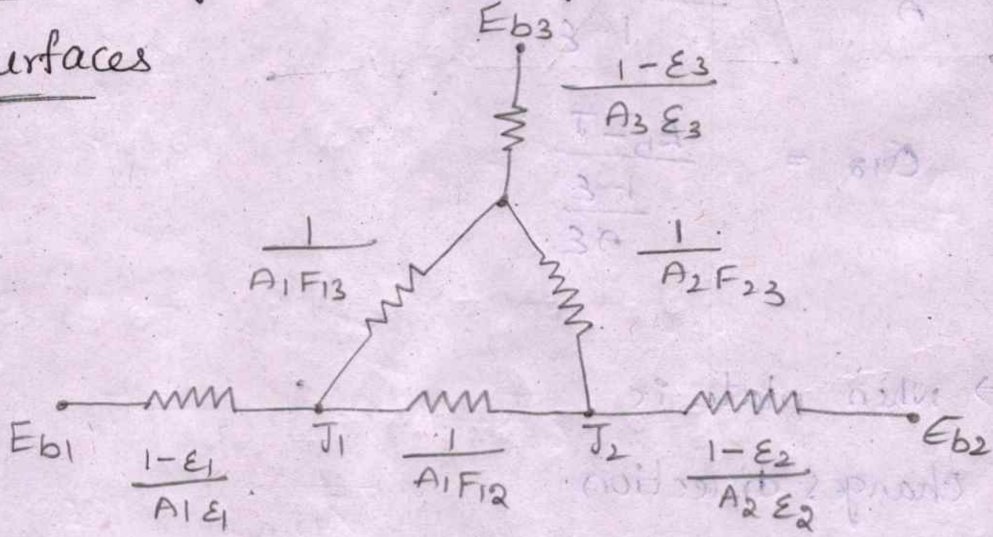
$$\varepsilon = \sigma T^4$$

For black bodies

$$Q_{12} = (E_{b1} - E_{b2}) \times A_1 F_{12}$$

$$\varepsilon_1 = \varepsilon_2 = 1$$

Radiation of Heat Exchange for Three Gray Surfaces



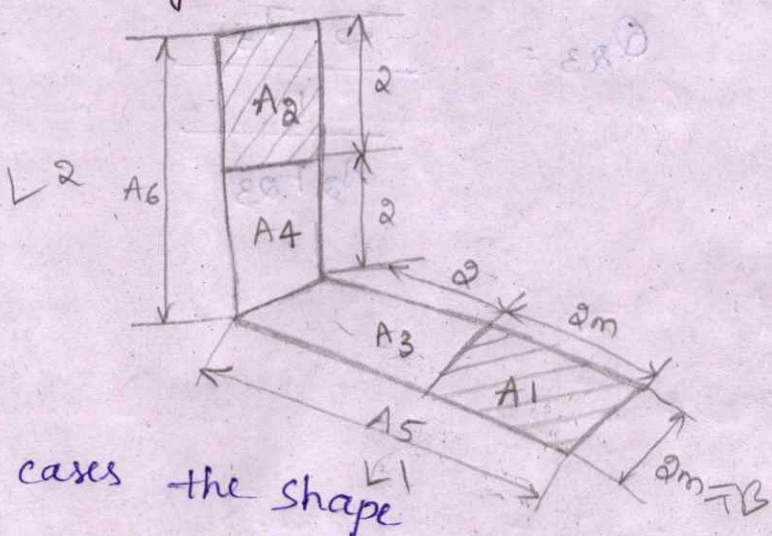
$$Q_{12} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}}$$

$$Q_{13} = \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}}$$

$$Q_{23} = \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$$

① Find the shape factor F_{12} for the figure shown below. In the figure the areas A_1 and A_2 are perpendicular but donot share the common edge.

Sol



⇒ for such cases the shape factor is evaluated by introducing hypothetical area. So, that the arrangement of perpendicular surface has a common edge.

Now $A_5 = A_1 + A_3$

$A_6 = A_4 + A_2$

$$A_5 F_{5-6} = A_1 F_{1-6} + A_3 F_{3-6}$$

$$= (A_1 F_{1-4} + A_1 F_{1-2}) + A_3 F_{3-6}$$

$$= \left[(A_5 F_{5-4} - A_3 F_{3-4}) + A_1 F_{1-2}\right] + A_3 F_{3-6} = A_5 F_{5-6}$$

$$\Rightarrow A_1 F_{12} = A_5 F_{5-6} - A_3 F_{3-6} - A_5 F_{5-4} + A_3 F_{3-4}$$

$$\Rightarrow A_1 F_{12} = (A_5 F_{5-6} + A_3 F_{3-4}) - (A_5 F_{5-4} + A_3 F_{3-6})$$

103 Pg

surface	z/x	y/x	shapefactor
A_{5-6}	$4/2 = 2$	$4/2 = 2$	0.14930
A_{3-4}	$2/2 = 1$	$2/2 = 1$	0.2004
A_{5-4}	$2/2 = 1$	$4/2 = 2$	0.11643
A_{3-6}	$4/2 = 2$	$2/2 = 1$	0.23285

$$Z = \frac{L_2}{B}$$

$$Y = \frac{L_1}{B}$$

$$A_1 = (2\times 2)\, F_{12} = 4x$$

$$A_5 = (4\times 2)\, F_{5-6} = 8 \times 0.14930 =$$

$$A_3 = (2\times 2)\, F_{3-4} = 4 \times 0.2004 =$$

A_6

$$F_{12} = (2\times 2) \times F_{1-2} = \left((4\times 2)\times 0.1491 + (2\times 2)\times 0.2004\right)$$

$$- \left((4\times 2)\times 0.11 + (2\times 2)\times 0.23\right)$$

$$F_{12} = 0.05$$

UNIT – III

Convective Heat Transfer

Dimensional Analysis

Dimensional analysis is a mathematical method which makes use of study of the dimensions for solving several engineering problems.

This method can be applied to all types of fluid resistances, heat flow problems & many other problems in fluid mechanics & thermodynamics.

Dimensions

In DA, the various physical quantities used these are expressed in fundamental quantities.

ex: mass (m), length (L), time (T), temp (θ)

The dimensions of commonly used quantities in heat transfer analysis is listed below.

M = Mass, θ = temp
L = Length, T = Time.

$$\text{velocity } V = \frac{\text{Distance}}{\text{Time}} = \frac{L}{T} = LT^{-1}$$

Quantity	Symbol	units (SI)	Dimensions (M, L, T, θ) system
Length	L	m	L
Area	A	m^2	L^2
velocity	V	m/s	LT^{-1}
acceleration	a, g	m/s^2	LT^{-2}
mass	m	kg	M
Density	ρ	kg/m^3	ML^{-3}
Force	F	N	MLT^{-2}

pressure	P	N/m^2	$ML^{-1}T^{-2}$
work	W	J	ML^2T^{-2}
Torque	T	J	ML^2T^{-2}
Power	P	W	ML^2T^{-3}
kinematic viscocity	ν	m^2/s	L^2T^{-1}
Discharge	Q	m^3/s	L^3T^{-1}
Heat	Q	J	ML^2T^{-2}
Heat transfer coefficient	h	W/m^2k	$MT^{-3}\theta^{-1}$
Thermal conductivity	K	W/mk	$MLT^{-3}\theta^{-1}$
thermal diffusivity	α	m^2/s	L^2T^{-1}
specific heat	C_p	J/kg k	$L^2T^{-2}\theta^{-1}$
Heat transfer rate	q	W	ML^2T^{-3}
coefficient of expansion	β	k^{-1}	θ^{-1}

Buckingham π Theorem

Buckingham's π theorem states that the no: of independent dimensionless Groups that can be obtained from a set of 'n' variables having r basic dimensions is $(n-r)$.

Reynolds Number (Re)

It is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho U^2 L^2}{\mu U L} = \frac{\rho U L}{\mu}$$

$$\boxed{Re = \frac{UL}{\nu}}$$

U = velocity, m/s

L = Length, m,

$\nu = \frac{\mu}{\rho}$ = Kinematic viscosity, m^2/s

Prandtl Number (Pr)

It is the ratio of the momentum diffusivity to the thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$$

ν = kinematic viscosity, m^2/s,

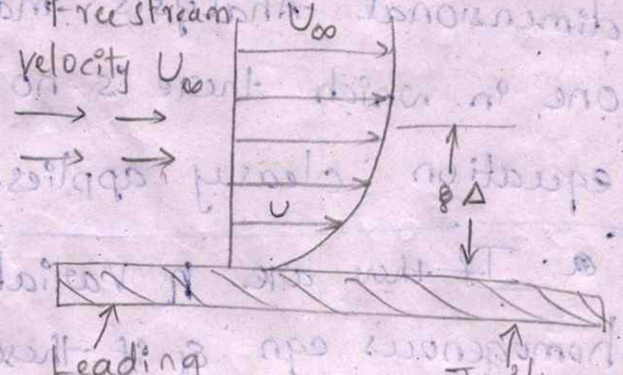
α = thermal diffusivity, m^2/s.

Nusselt Number (Nu)

It is defined as the ratio of the heat flow by convection process under an unit temperature gradient

$$Nu = \frac{hL}{k}$$

Newtonian Non Newtonian, Laminar Turbulent Boundary Layer Concept



Boundary layer on flat plate

Types of Boundary Layer

1. Hydrodynamic boundary layer
(or)
Velocity boundary layer

2) Thermal boundary layer.

Hydrodynamic Boundary Layer

In hydrodynamic boundary layer, velocity of the fluid is less than 99% of free stream velocity.

Thermal boundary layer

In thermal boundary layer, temperature of the fluid is less than 99% of free stream temperature

Types of convection:

1) Free convection 2) Forced convection

Free (or) Natural convection

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free & natural convection.

Forced Convection

If the fluid motion is artificially created by means of an external ~~force~~ like a blower or fan that type of heat transfer is known as forced convection.

11/05/2023

1) Air at $20°C$, at a pressure of 1 bar flowing over a flat plate at a velocity of 3 m/sec. If the plate is maintain at $60°C$, Calculate the heat transfer per unit width of the plate. Assuming the length of the plate along the flow of air is 2m.

Sol

$T_\infty = 20°C$

$P = 1$ bar.

$u = 3$ m/sec

$T_w = 60°C$

width $w = 1$ m

$L = 2$m

$$T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{60+20}{20} = 40°C$$

from data book Propeities of Gas & Vapour (42)

$\rho = 1.128$ kg/m³

$K = 0.02756$ W/mK.

$P_r = 0.699$

$\nu = 16.96 \times 10^{-6}$ m²/sec

$$Re = \frac{uL}{\nu} = \frac{3 \times 2}{16.96 \times 10^{-6}} = 353 \times 10^{3} = 3 \times 10^{5}$$

∴ it is less than $5 \times 10^5 < 3 \times 10^5$

it is a laminar flow.

Nusselt number $Nu = \frac{hL}{K} =$

$$Nu = 0.332\, Re_x^{0.5}\, P_r^{0.333}$$

$$= 0.332 \times (3\times10^5)^{0.5} \times (0.699)^{0.333}$$

$$= 156.54$$

$$\Rightarrow h = \frac{Nu \cdot k}{L} = \frac{156.5 \times 0.02756}{2} = 2.157$$

$$\Rightarrow Q = hA(T_w - T_\infty)$$

$$Q = 2.157 \times (2\times1)(60-20)$$

$$= 172.52\, W \times 2$$

$$= 345.05\, W$$

$$h = 2\times h_x$$
$$= 2\times172.5$$
$$=$$

(2) Air at 20°c at atmospheric pressure flows over a plate at a velocity of 30 m/sec. If the plate is 1m wide and 80°c. calculate the following at $x = 300mm$ =

i) hydrodynamic boundary layer thickness.

ii) Thermal boundary layer thickness.

iii) Local friction coefficient

iv) avg. friction coefficient

v) Local heat transfer coefficient

vi) Avg. heat transfer coefficient

vii) Heat transfer.

Sol Given data

$T_\infty = 20^\circ C$

$u = 30 m/sec$

$W = 1m$

$T_w = 80^\circ C$

$x = 0.3m$ = Length.

$T_f = \frac{T_w + T_\infty}{2} = \frac{20+80}{2} = 50^\circ C$

(i) hydrodynamic boundary layer thickness.

taking values from data book: at 50°C (properties)

$\rho = 1.093 \ kg/m^3$

$\nu = 17.95 \times 10^{-6} \ m^2/sec$

$Pr = 0.698$

$K = 0.02826 \ W/mK$.

$\delta_{hx} = 5x \ Re_x^{-0.5}$

$= 5 \times 0.3 \times (5 \times 10^4)^{-0.5}$

$= 6.7 \times 10^{-3} \ m.$

$Nu = \frac{hL}{K} =$

$Re = \frac{UL}{\nu} = \frac{30 \times 0.3}{17.95 \times 10^{-6}} = 5 \times 10^4$

it is laminar flow.

(ii) Thermal boundary layer thickness

$$\Delta_{Tx} = \Delta_{hx} \cdot P_r^{-0.333}$$

$$= 6.7 \times 10^{-3} \times (0.698)^{-0.333}$$

$$= 7.56 \times 10^{-3} \text{ m}$$

(iii) Local friction coefficient

$$C_{fx} = 0.664 \, Re_x^{-0.5}$$

$$= 0.664 \times (5 \times 10^4)^{-0.5}$$

$$= 2.96 \times 10^{-3}$$

(iv) Avg. friction coefficient

$$C_{fL} = 1.328 \, Re_L^{-0.5}$$

$$= 1.328 \times (5 \times 10^4)^{-0.5}$$

$$= 5.9 \times 10^{-3}$$

(v) Local heat transfer coefficient

$$Nu = \frac{hL}{k}$$

$$Nu = 0.332 \times Re^{0.5} \times P_r^{0.333}$$

$$= 65.86$$

$$h_x = \frac{65.86 \times 0.02826}{0.3} = 6.2 \text{ W/m}^2\text{k}$$

⑥ Avg. heat transfer coefficient (h)

$h = 2 \times h_x = 12.4 \ w/m^2k$

$h = 6.2 \times 2 = 12.4 \ w/m^2k.$

⑦ Heat transfer (Q).

$Q = h \times A (T_w - T_\infty)$

$A = W \times L = 1 \times 0.3 = 0.3$

$Q = 223.38 W.$

$= 12.4 \times 0.3 (80 - 20)$

$\boxed{Q = 223.2 W}$

③ Air at $30^\circ C$ flows over a flat plate at a velocity of 4 m/sec. The plate is maintain at $90^\circ C$. The plate dimension $90 \times 30 \ cm^2$ calculate the heat transfer for following condition.

(i) ~~first~~ half half the plate

(ii) full plate

(iii) Next half half the plate

Sol

$T_\infty = 30^\circ C$

$T_w = 90^\circ C$

$u = 4 \ m/sec.$

Plate $= 90 \times 30 \ cm^2$

$= \underset{L}{0.9} \times \underset{W}{0.3} \ m^2$

$T_f = \frac{T_w + T_\infty}{2} = \frac{30 + 90}{2} = \frac{120}{6} = 60^\circ C$

$\rho = 1.060 \ kg/m^3$

$k = 0.02896 \ W/mK$

$\nu = 18.97 \times 10^{-6} \ m^2/sec$

$P_r = 0.696$

1 $x = L = 0.9 m$

$x = \frac{L}{2} = \frac{0.9}{2} = 0.45 m.$

$Re = \frac{uL}{\nu} = \frac{4 \times 0.45}{18.97 \times 10^{-6}} = 0.9 \times 10^5$

$Nu = 0.332 \times R_e^{0.5} \times P_r^{0.333}$

$= 0.332 \times (0.96 \times 10^5)^{0.5} \times (0.696)^{0.333}$

$= 91.17$

$Nu = \frac{hL}{k}$

$h_x = \frac{91.17 \times 0.02896}{0.45} = 5.86 \ W/m^2k.$

$h = 2 \times 5.86 = 11.73 \ W/m^2k.$

$Q_1 = hA (T_w - T_\infty)$

$= 11.73 \times 0.135 (90 - 30)$

$\boxed{Q_1 = 95.013 W}$

$A = 0.45 \times 0.3$
$= 0.135$

(ii) full plate

$L = 0.9$

$$Re = \frac{uL}{V} = \frac{4 \times 0.9}{18.97 \times 10^{-6}} = 1.89 \times 10^{5}$$

$$Nu = 0.332 \times Re^{0.5} \times Pr^{0.333}$$

$$= 0.332 \times (1.89 \times 10^{5})^{0.5} \times (0.696)^{0.333}$$

$$= 127.92$$

$$\Rightarrow Nu = \frac{hL}{k}$$

$$h_x = \frac{Nu\,k}{L} = \frac{127.05 \times 0.02896}{0.9} = 4.11$$

$$h = 4.11 \times 2 = 8.22 \ W/m^2 k.$$

$$Q_2 = hA(T_w - T_\infty)$$

$$= 8.22 \times 0.27 (90 - 30)$$

$$Q_2 = 133.16 \ W$$

$A = 0.9 \times 0.3$
$= 0.27 \ m^2$

(iii) $Q = Q_2 - Q_1$

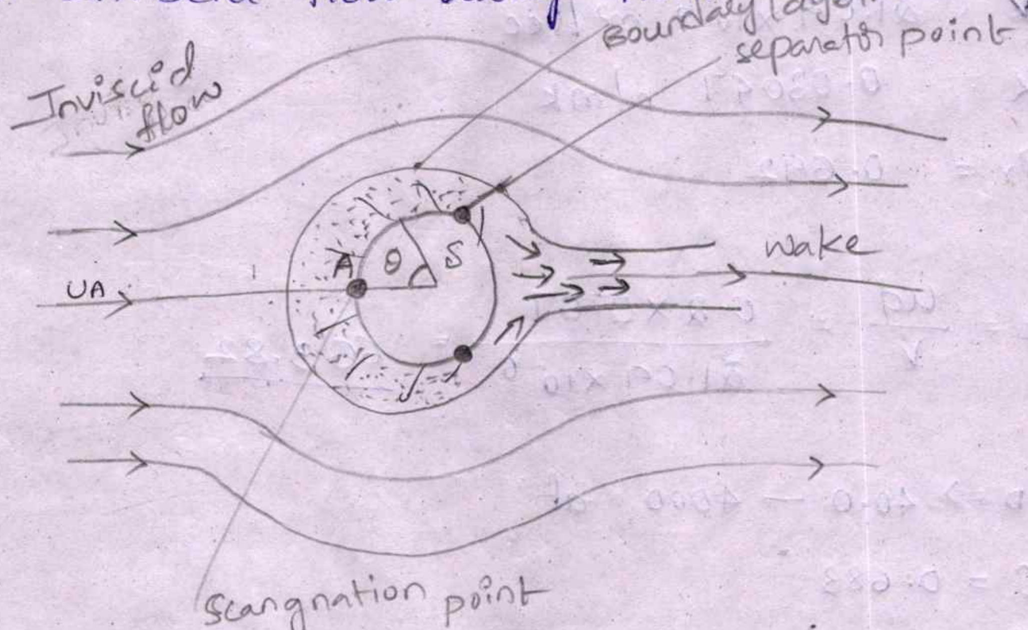
$$= 133.16 - 95.013$$

$$Q = 38.15 \ W$$

Flows over a Cylinder & Sphere

The flow field can be divided into two regions. They are

(i) Boundary layer region near the surface
(ii) Inviscid flow away from the surface



① Air at 30°, 0.2 m/sec flows across a 120W electric bulb at 130°C find heat transfer & Power lost due to Convection. if bulb diameter is 70mm.

sol Given

fluid temp $T_\infty = 30^\circ C$

velocity $U = 0.2$ m/sec

heat energy $Q_1 = 120W$

Surface temp $T_w = 130^\circ C$

dia $D = 70$ mm $= 0.07$m.

$$T_f = \frac{T_w + T_\infty}{2} = \frac{30+130}{2} = \frac{160}{2} = 80^\circ C$$

propess of air

$\rho = 1.000 \ kg/m^3$

$\nu = 21.09 \times 10^{-6} \ m^2/sec$

$K = 0.03047 \ W/mK$

$P_r = 0.692$

$$Re = \frac{ud}{\nu} = \frac{0.2 \times 0.07}{21.09 \times 10^{-6}} = \underline{663.82}$$

$\times$ ($Re_D \Rightarrow 40.0 - 4000$ at

$C = 0.683$

$m = 0.466$) $\times$

$\times$ ($Nu_D = C \cdot Re_D^{\ m} \cdot P_r^{\ 0.333}$

$= 0.683 \times (663.82)^{0.466} \times (0.692)^{0.333}$) $\times$

$\Rightarrow Nu = 0.37 \cdot Re^{0.6}$

$= 0.37 \times (663.8)^{0.6}$

$= 18.25$

$$\boxed{\begin{aligned} A &= 4\pi r^2 \\ &= 4\pi \times 0.035^2 \\ &= 0.0153 \end{aligned}}$$

$$Nu = \frac{hD}{K}$$

$$h = \frac{18.25 \times 0.03047}{0.07} = 7.94$$

$$Q = 7.94 \times 0.0153 \times (130-30)$$

$$\boxed{Q_2 = 12.14 \, W}$$

(ii) % of heat lost due to convection.

$$= \frac{Q_2}{Q_1} \times 100$$

$$= \frac{12.14}{120} \times 100 = 10.11$$

(2) Air at 40°C flows over a tube with a velocity of 30 m/sec. The tube surface temp is 120°C. Calculate the heat transfer coefficient for the following cases.

(i) tube is considered to be square of side (6cm)

(ii) tube is circular cylinder of diameter 6cm.

Sol)

$T_\infty = 40°C$

$T_w = 120°C$

$U = 30$ m/sec

$$T_f = \frac{T_w + T_\infty}{2} = \frac{40+120}{2} = \frac{160}{2} = 80°C$$

air =

$\rho = 1.000$ kg/m³

$\nu = 21.09 \times 10^{-6}$ m²/sec

$K = 0.03047$ W/mK

$P_r = 0.692$

(i) $h = ?$

$L = 6cm = 0.06m$

$$R_e = \frac{UL}{\nu} = \frac{30 \times 0.06}{21.09 \times 10^{-6}} = 85 \times 10^3$$

(i) $Nu = 173.3$

$h = 88$ (127, c, n)

(ii) $Nu = 219.3$

(124) $h = 111.3$

$Re = 0.85 \times 10^5$

$\Rightarrow N = C Re^{n} = 0.092(0.85\times10^{5})^{0.675}$

$= 195.4$

$= 1.13\, P_r^{0.33} = 1.13\times(0.692)^{0.33}$

$\Rightarrow h = \dfrac{Nu\, k}{L} \qquad = 195.54$

(ii) Flow over cylinder (124 pa)

$Nu = C Re_D^{m} \cdot P_r^{0.33} \qquad C = 0.0266, \quad 0.805 = m$

$= 0.0266\times(0.85\times10^{5})^{0.805}\times(0.692)^{0.33}$

$= 219.3$

$Nu = \dfrac{hD}{K} = 111.3\, W/m^2K.$

Flows over Banks of Tubes

① In a surface condenser, water flows through inline Tubes. while the air is passed in cross-flow over the tubes. The temp and velocity of air are 30°c & 8 m/sec. Respectively The longitudinal and Transfer pitch are 22mm and 20mm respectively. The tube outside diameter is 18 mm & Tube surface temp is 90°c.

Sol Given

$T_\infty = 30°C$

$u = 8\, m/s$

$S_L = 22\, mm = 0.022\, m$

$S_T = 20\, mm = 20\times10^{-3}\, m.$

$d = 18\, mm = 0.018\, m.$

$T_w = 90°C$

$T_f = \frac{T_w + T_\infty}{2} = 40°C \longrightarrow \rho, k, \nu, P_r$

$Nu = \frac{hd}{k} = h = ?\ 426.6\ W/m^2K.$

$Nu = \cancel{0.33\times}\ 1.13 \times P_r^{0.33} \times C.Re^n$. [131 Pg]

$=$

$\Rightarrow Re = \frac{ud}{\nu} = 7.5\times10^4 = \frac{80 \times 0.018}{18.97\times10^{-6}}$.

Inline [129 Pg]

$U_{max} = \left[\frac{St}{St - D}\right] u_\infty$

$= \left(\frac{20\times10^{-3}}{20\times10^{-3} - 0.018}\right) \times 8$

$= 80\ m/sec$

$\frac{St}{D} = 1.11$

$\frac{Sl}{D} = 1.22$

$\left.\begin{matrix} \\ \end{matrix}\right\} \quad C = 0.348,\ n = 0.592$

Internal Flow: Flow through a Cylinder

① lubricating oil at a temperature of 60°c enters 1 cm diameter tube with a velocity of 3 m/sec. The tube surface is maintain at 40°c. Assuming that the oil has the following average properties, calculate the tube length required to cool the oil at 45°c.

$\rho = 865 \ kg/m^3$, $k = 0.140 \ W/mK$

$C_p = 1.78 \ kJ/kg°c$

$1.78 \times 10^3 \ J/kg°c$

Sol Tmi $= 60°c$

$D = 1 \ cm = 0.01 m$ $L = ?$

$U_m = 3 \ m/sec$

$T_w = 40°c$

$T_{mo} = 45°c$

$\Rightarrow Q = hA\Delta T$

$\Rightarrow Q = mc_p \Delta T$

$m = \rho \times A \times u$

$= \ \frac{\pi}{4}d^2$

$= 0.204 \ m/sec$

$\Rightarrow Nu = 3.66 = \frac{hd}{k}$

$h = 51.24 \ W/m^2K.$

$A = \pi \times d \times L \rightarrow 270.6\ m$

$Q = hA\Delta T$

$\Delta T = \left(\frac{T_m + T_i}{2}\right) - T_w$

$Q = mc_p(T_m - T_{T_i})$

$= 56.2 \times 4178 (50-30)$

$= 4.69 \times 10^6 W$

$Q = hA(T_w - T_m)$

$A = \pi DL$

$L = ?$

Free Convection

due to temp. difference, density changed in the fluid flow.

① A vertical plate of 0.75m height is at 170°C and is exposed to air at a temp of 105°C and 1 atmospheric pressure. Calculate

(i) Mean heat transfer coefficient

ii) rate of heat transfer per unit width

Sol) $L = 0.75m = x$

$T_w = 170°C$

$T_\infty = 105°C$

$T_f = \frac{T_w + T_\infty}{2} = 137.5 \cong 140°C$

$\rho = 0.854 \ kg/m^3$

$\nu = 27.80 \times 10^{-6} \ m^2/sec$

$Pr = 0.684$

$K = 0.03489 \ W/mK.$

Gr.Pr

$Gr = \frac{g \times \beta \times x^3 \ \Delta T}{\nu^2} = \cancel{2.5 \times 10^9} \ 835 \times 10^6$

$\beta = \frac{1}{T_f} = \frac{1}{(137.5 + 273)} = \cancel{7.27 \times 10^{-3}} = 2.4 \times 10^{-3}$

$\cancel{Gr.Pr = 2.5 \times 10^9 \times 0.684}$

$Gr.Pr = 8.3 \times 10^8 \times 0.684$

$Gr.Pr = 5.7 \times 10^8$

$Gr.Pr < 10^9$

it is laminar flow.

$\Rightarrow Nu = 0.59 (Gr Pr)^{0.25}$ → 194 Pg. higher values.

$= 0.59 (5.7 \times 10^8)^{0.25}$

$= 91.16$

$\Rightarrow Nu = \frac{hL}{K}$

$h = \frac{91.16 \times 0.03989}{0.75}$

$= 4.24 \ W/m^2 K$

$\Rightarrow Q = hA (T_w - T_\infty)$

$= 4.24 \times 0.75 (170 - 105)$

$= 206.7 \ W$

$A = W \times L$
$= 1 \times 0.75$
$= 0.75 m^2$

(2) A thin 100cm long and 10cm wide horizontal plate is maintained at a uniform temperature of 150°C in a large tank of water at 75°C. Estimate the rate of heat to be supplied to the plate maintain constant plate temp as heat is dissipated either side of plate

Sol. Given data $\boxed{x = W}$ while cal at Gr

$l = 100\,cm = 1\,m$

$W = 10\,cm = 0.1\,m$ = horizontal position.

$T_w = 150°C$

$T_\infty = 75°C$

$T_f = \dfrac{T_w + T_\infty}{2} = \dfrac{150+75}{2} = \dfrac{225}{2} = 112.5°C$

at 100°c

$\rho = 961\ kg/m^3$

$\nu = 0.293 \times 10^{-6}\ m^2/sec$

$Pr = 1.740$

$k = 0.6804\ W/mK$

$\beta = \dfrac{1}{(T_f + 273)} = \dfrac{1}{(112.5+273)} = 2.5 \times 10^{-3}$

$Gr = \dfrac{g \cdot \beta \times x^3\, \Delta T}{\nu^2} = \dfrac{9.81 \times 2.5 \times 10^{-3} \times \left(\frac{0.1}{2}\right)^3 \times (75)}{(0.293 \times 10^{-6})^2}$

$x = \dfrac{W}{2}$ only for finding Gr value. $= \dfrac{0.1}{2} = 0.05$

$Gr = 2.67 \times 10^9$

$Gr\,Pr = 2.67 \times 10^9 \times \cancel{2.6}\,1.740.$

$= 4.6 \times 10^9$

$$\Rightarrow Nu = 0.15\,(GrPr)^{0.333}$$

$$= 247.62$$

$$\Rightarrow Nu = \frac{hL}{K}$$

$$h = \frac{247.62 \times 0.6804}{0.05}$$

$$\Rightarrow h_1 = 3369.6\ W/m^2K.$$

$$\Rightarrow Nu = 0.27 \times (GrPr)^{0.25}$$

$$=$$

$$= 70.31$$

$$\Rightarrow h_2 = \frac{70.31 \times 0.6804}{0.05} = 956.77\ W/m^2K.$$

$$Q = hA(\Delta T)$$

$$= 3439.9 \times 0.1 \times (75)$$

$$Q = 25799.25\ W$$

$$h = h_1 + h_2$$
$$= 3439.91$$

$$A = 0.1 \times 1$$
$$= 0.1\,m^2$$